SANGSTER'S

NATURAL PHILOSOPHY

PART. I.

(4000)

$\frac{1}{12} = \frac{3}{36} = \frac{8}{432}$

early Canadian science

LOVELL'S SERIES OF SCHOOL BOOKS.

NATURAL PHILOSOPHY,

PART I,

INCLUDING

STATICS, HYDROSTATICS, PNEUMATICS, DYNAMICS, HYDRODYNAMICS, THE GENERAL THEORY OF UNDULATIONS, THE SCIENCE OF SOUND, THE MECHANICAL THEORY OF MUSIC, ETC.

DESIGNED

FOR THE USE OF NORMAL AND GRAMMAR SCHOOLS, AND THE HIGHER CLASSES IN COMMON SCHOOLS.

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PREFACE TO FIRST EDITION.

The following Treatise was originally designed to serve as a hand-book or companion to the lectures on Natural Philosophy, delivered to the junior division in the Normal School. Although numerous text-books on the subject were already in existence, it was found that they were either too abstruse and technical for beginners, or too general and superficial to be of much practical use. The aim of the present little work is to occupy a position between these extremes—to present the leading facts of the science in a form so concise as to be readily remembered, and at the same time to give that thorough drilling upon the principles which is absolutely essential to their full comprehension.

As a hand-book to lectures fully illustrated by apparatus, it was not necessary to introduce many wood-cuts, and accordingly they have been given only where absolutely required.

The chief peculiarity of this book consists in the introduction, to a large extent, of problems calculated to impart that intimate and practical knowledge of the facts and principles of Mechanical Science, without which the student's information on the subject is, comparatively speaking, useless. How frequently do we meet with a pupil who has read carefully through one of the common text books on Natural Philosophy without acquiring any very clear or definite ideas of the science! And what should we say of a work professing to teach the principles of arithmetic or algebra by mere rules and explanations, without an appropriate selection of examples and problems? The exercises are therefore deemed an important feature of the following pages, and it is thought that the science may be taught by their aid more thoroughly and in less time than otherwise.

TORONTO, January, 1860.

PREFACE TO SECOND EDITION.

The proof sheets of this edition have undergone the most attentive revision at the hands of the Author. He has added a section on the Turbine Water Wheel, a chapter on the Theory of Uudulations, another on the Science of Sound, and a third on the Mechanical Theory of Music. The Author trusts that these additions will render the work more serviceable and more deserving of that flattering reception which has been already accorded to it.

TORONTO, February, 1861.

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NATURAL PHILOSOPHY.

CHAPTER I.

SUBDIVISIONS—GENERAL PROPERTIES OF MATTER—ATTRACTION.

1. Natural Science, in its widest sense, embraces the study of all created objects and beings, and the laws by which they are governed.

2. Natural objects are divided into two great classes, viz., *organic* and *inorganic*, the former being distinguished from the latter by the exhibition of *vital power* or *life.*

3. Organic existences are separated into *animals* and *vegetables*, the former distinguished from the latter by the possession of *sensibility* and *volition.*

4. The different subdivisions of natural science and their objects are as follows :—

Zoology describes and classifies *animals.*

Botany teaches the classification, use, habits, structure, &c., of *plants.*

Mineralogy describes and classifies the various *mineral constituents* of the earth's crust.

Astronomy investigates the laws, &c., of *celestial phenomena.*

Geology has for its object the description, &c., of the *crust of the earth.*

Chemistry teaches us how to *unite* two or more *elementary bodies* into one compound, or how to *decompose compound bodies* into their simple elements.

Natural Philosophy or Physics has for its object the investigation of the *general properties* of all bodies, and the *natural laws* by which they are regulated.

5. Natural Philosophy is divided into—

I. General Mechanics—including Statics, Hydrostatics, Dynamics, Hydrodynamics, and Pneumatics.

II. Heat.

III. Light—including Perspective, Catoptrics, Dioptrics, Chromatics, Physical Optics, Polarization, and Actino-Chemistry.

IV. Electricity—including Statical Electricity, Galvanism, Magnetism, Thermo-Electricity, and Animal Electricity.

V. Acoustics.

PROPERTIES OF MATTER.

6. Matter exists in three separate forms,—I. *Solid;* II. *Liquid;* and III. *Gaseous.*

NOTE.—The same body may exist in all three forms, as is the case with *water, mercury, sulphur,* &c. The amount of heat or caloric present determines the form of the body—if heat be applied, the attraction of cohesion existing among the particles is gradually overcome, and the body passes from a solid to a liquid, and from a liquid to a gas. If heat be abstracted, the attraction of cohesion gradually draws the particles into closer proximity, and the body passes from a gas to a liquid, and finally from a liquid to a solid. Hence heat and cohesion are called *antagonistic forces.*

7. Matter is distinguished by the possession of certain distinctive properties.

8. The properties of matter are divided into—

1st. Essential Properties.
2nd. Accessory Properties.

9. The *essential* properties of matter are those without which matter could not possibly exist.

10. The essential properties of matter are *Extension, Impenetrability, Divisibility, Indestructibility, Porosity, Compressibility, Inertia, and Elasticity.*

11. *Extension* implies that every body must fill a certain portion of space.

NOTE.—The Dimensions of Extension are *length, breadth,* and *thickness.*

12. *Impenetrability* implies that no two bodies can occupy the same portion of space at the same time.

NOTE—Examples of the impenetrability of matter will readily suggest themselves. Among the more common may be mentioned the impossibility of filling a bottle with water until the air is displaced—the fact that when the hand is plunged into a vessel *filled* with water, a portion of the liquid overflows, &c. All instances of the apparent penetrability of matter are merely examples of displacements. Thus, when a nail is driven into a piece of wood, it *displaces* the particles of wood, driving them closer together.

13. *Divisibility* is the capability of being continually divided and subdivided, and is an essential property only of *masses* of matter.

NOTE 1.—The ultimate particles of matter ; i. e., those inconceivably minute molecules which cannot be further subdivided, are termed *atoms*. (*Gr. a* " not" and *temno* " to cut ;" i. e., that which cannot be cut or divided.)

NOTE 2.—The following may be given as examples of the extreme divisibility of matter :—

I. Gold leaf is hammered so thin that 300,000 leaves placed one on another, and pressed so as to exclude the air, measure but *one inch* in thickness.

II. Wollaston's micrometric wire is so fine that 30,000 wires placed side by side, measure but *one inch* across—150 of these wires bound together do not exceed the diameter of a filament of raw silk, 1 mile of the wire weighs but a grain, and 7 ounces would reach from Toronto to England.

III. Insects' wings are some of them so fine that they do not exceed the $\frac{1}{200000}$ of an inch in thickness.

IV. The thinnest part of a soap bubble is only the 2,500,000th part of an inch in thickness.

V. Blood corpuscles are so small that it requires 50,000 corpuscles of human blood, or 800,000 corpuscles of the blood of the musk-deer to cover the head of a common pin. Yet these corpuscles are compound bodies, and may be resolved, by means of chemistry, into their simple elements.

VI. There are animalcules so minute that millions of them heaped together do not equal the bulk of a single grain of sand, and thousands might swim side by side through the eye of the finest cambric needle. Yet these creatures possess, in many cases, complicated organs of locomotion, nutrition, &c.

VII. At Bilin, in Bohemia, a huge mountain consists entirely of shells, so minute, that a cubic inch contains 41 billions—a number so vast that counting as rapidly as possible day and night without intermission, it would require 780 years to enumerate it.

VIII. The filament of the spider's web is so fine that 4 miles of it weigh only about a grain—yet this thread is formed of about 6000 filaments united together, &c., &c.

14. *Indestructibility* implies that it is as impossible for a finite creature to annihilate as to create matter.

NOTE.—We can change the form of matter at pleasure, but we cannot destroy it. When fuel, for example, is burned, not a particle is lost, as is proved by the fact that if we collect all the products of the combustion ; i.e., the smoke, soot, ashes, &c., and weigh them, we shall find their aggregate weight exactly equal to that of the wood or coal consumed. We may safely conclude that there is not a single atom of matter, more or less, attached to our earth now than at the time of Adam.

15. *Porosity* implies that the constituent atoms of matter do not touch each other, but are separated by small intervening spaces called *pores*.

NOTE.—The atoms even of the densest bodies are much smaller than the spaces which separate them. Newton regards them as *infinitely smaller*, as being in fact mere mathematical points; and Sir J. Herschel asks why the particles of a solid may not be as thinly distributed through the space it occupies as the stars that compose a nebula, and he compares a ray of light penetrating glass to a bird threading the mazes of a forest.

16. *Compressibility* implies the capability a body possesses of being forced into a smaller bulk without any diminution in the *quantity of matter* it contains.

NOTE.—Since all matter is porous, it follows, as a necessary consequence, that all matter must be compressible.

17. *Inertia* means passiveness or inactivity, or that matter is incapable of changing its state, either from rest to motion or from motion to rest.

NOTE.—Bodies moving on or near the surface of the earth soon come to a state of rest, unless some constant propelling force is applied to them. This is owing to the action of certain *resisting* forces, as the resistance of the atmosphere, friction, and the attraction of gravity.

18. *Elasticity* is the capability which all bodies possess, more or less, of recovering their former dimensions after compression, or after having, for a time, been compelled to assume some other form.

NOTE.—As applied to solids, elasticity is divided into—
1. Elasticity of compression,
2. Elasticity of tension,
3. Elasticity of flexure, and
4. Elasticity of torsion.

Some bodies, as putty, seem to possess very little elasticity. In glass all four kinds appear to exist almost perfect within certain limits—no force however great or long continued will cause glass to take a *set* as it is termed.

19. The *accessory* properties of matter are those which merely serve to distinguish one kind of matter from another.

20. The *accessory* properties of matter are *hardness, softness, flexibility, brittleness, transparency, opacity, malleability, ductility, tenacity, &c.*

21. *Malleability* expresses the susceptibility, possessed by certain kinds of matter, of being *hammered out* into thin sheets.

NOTE.—The most malleable metals are gold, silver, iron, copper, and tin.

22. *Ductility* is susceptibility of being *drawn out* into fine wire.

NOTE.—The most ductile metals are platinum, gold, iron, and copper.

23. *Tenacity* or toughness implies that a certain force is necessary to pull the particles of a body asunder.

NOTE.—The following table shows the relative tenacity of different substances. The first column shows the number of pounds weight required to tear asunder a prism of each substance having a sectional area of *one square inch*, and the second column gives the length of the rod of any given diameter, which, if suspended, would be torn asunder by its own weight:—

TABLE OF TENACITY.

	Weight in pounds. (Section of rod 1 sq. in.	Length in feet, (any diameter.)
	METALS.	
Cast lead.	1824	348
Cast tin,	4736	1496
Yellow brass,	17958	5180
Cast copper,	19072	5003
Cast iron,	19096	6110
English malleable iron,	55872	16938
Swedish do.	72064	19740
Cast steel,	134256	39455
	WOODS.	
Pine,	9540	40500
Elm,	9720	35800
Oak,	11880	32900
Beech,	12225	38940
Ash,	14130	42080

ATTRACTION.

24. *Attraction* is that power in virtue of which particles and masses of matter are drawn towards each other.

25. Attraction is of several kinds, viz:

I. Attraction of Gravity.
II. Attraction of Cohesion.
III. Attraction of Adhesion.
IV. Capillary Attraction.
V. Electrical Attraction.
VI. Magnetic Attraction.
VII. Chemical Attraction.

26. *Attraction of Gravity* (Lat. *gravitas*, "weight") is that force by which *masses of matter* tend to approach each other. It is sometimes spoken of as *gravitation*, or when applied to the force by which bodies are drawn towards the centre of the earth, *terrestrial gravity*.

27. The intensity of the force of gravity varies directly as the mass of the bodies, and inversely as the square of their distance apart.

NOTE.—If we suppose two spheres of any kind of matter, lead, for example, to be placed in presence of each other and under such conditions that being themselves free to move in any direction they are entirely uninfluenced by any other bodies or circumstances, they will approach each other, and;—

1st. If their masses are equal, their velocities will be equal.

2nd. If one contain twice as much matter as the other, its velocity will be only half as great as that of the other.

3rd. If one be infinitely great in comparison with the other, its motion will be infinitely small in comparison with that of the other; and

4th. The more nearly they approach each other, the more rapid will their motion become:—

28. By saying the *intensity* of the force of gravitation *varies inversely as the square* of the distance between the attracting bodies, we merely mean that if the attractive force exerted between two bodies at any given distance apart be represented by the unit 1, then, if the distance, apart be doubled, the force of attraction will be reduced to $\frac{1}{4}$ of what it was before; if the distance between the bodies be increased to three times what it was, the force of gravity will be decreased 9 times, or will be only $\frac{1}{9}$ of what it was, &c.

EXAMPLE 1.—If a body weigh 981 lbs. at the surface of the earth, what will it weigh 8000 miles from the surface?

SOLUTION.

Here since the distance of the body in the first case is 4000 miles from the centre of the earth and in the latter case 12000 (i. e. 8000 + 4000) the distance apart has been trebled.

Then weight $= \frac{981}{3^2} = \frac{981}{9} = 109$ lbs. *Ans.*

EXAMPLE 2.—The moon is 240,000 miles from the (centre of) earth, and is attracted to the earth by a certain force. How much greater would this force become if the moon were at the surface of the earth?

SOLUTION.

Here $\frac{240000}{\text{Earth's radius}} = \frac{40}{240000} = 60$, and $60^2 = 3600$ times. *Ans.*

EXERCISE.

3. If a mass of iron weigh 6700 lbs. at the surface of the earth, how much would it weigh at the distance of 12,000 miles from the surface? *Ans.* 418¾ lbs.

4. If a piece of copper weigh 9 lbs, at the distance of 36,000 miles from the earth's surface, what would it weigh at the surface of the earth? *Ans.* 900 lbs.

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29. *Attraction of Cohesion* is that force by which the constituent particles of the same body are held together.

NOTE.—The attraction of cohesion acts only at insensible distances; i. e., at distances so minute as to be incapable of measurement. The attraction of gravity, on the other hand, acts at sensible distances.

30. *Attraction of Adhesion* is that force by which the particles of dissimilar bodies adhere or stick together.

31. *Capillary Attraction* (*Lat. capilla*, "a hair") is the force by which fluids rise above their level in confined situations, such as small tubes, the interstices of porous substances, &c.

NOTE—It is by capillary attraction that oil and burning fluid, melted tallow, &c., rise up the wick of a lamp or candle.

32. *Electrical Attraction* is the force developed by friction on certain substances, as glass, amber, sealing-wax, &c.

33. *Magnetic Attraction* is the force by which iron, nickel, &c., are drawn to the loadstone.

34. *Chemical Attraction*, or Chemical Affinity, is the force by which two or more dissimilar bodies unite so as to form a compound essentially different in its appearance and properties from either of its constituents.

Thus Potash and Grease unite to form soap—Sulphur and Mercury unite to form Vermillion, &c.

CHAPTER II.

STATICS.

35. The Science of general mechanics (Greek *mèchanè*, "a machine") has for its object the investigation of the action of forces on matter whether they tend to keep it at rest or to set it in motion.

36. The Science of general mechanics is usually subdivided as follows:—

I. STATICS, (Greek *statos*, "standing") or the science by which the conditions of the *equilibrium of solids* are determined.

II. HYDROSTATICS, (Greek *hùdor*, "water," and statos, "standing,") or the science by which the conditions of the *equilibrium of liquids* are determined.

III. DYNAMICS, (Greek *dùnamis*, "force") or the science by which the laws that determine the *motions of solids* are investigated.

IV. HYDRODYNAMICS (Greek *hùdor* and *dùnamis*) or the science by which the laws that determine the *motions of liquids* are investigated.

V. PNEUMATICS (Greek *pneuma*, "air," and *statos*, "standing,") or Pneuma-statics, the science by which the conditions of the *equilibrium of elastic fluids*, as atmospheric air, are investigated. Pneumatics may be regarded as a branch of Hydrostatics.

37. A body is said to be in equilibrium when the forces which act upon it mutually counterbalance each other or are counterbalanced by some passive force or resistance.

38. Forces that are balanced so as to produce rest are called *statical forces* or *pressures*, to distinguish them from *moving, deflecting, accelerating* or *retarding forces.*

39. A force has three elements, viz., *magnitude, direction*, and *point of application.*

40. A force may be represented either by saying it is equal to a certain number of lbs., oz., &c., or by a line of definite length. A line has the advantage of completely defining a force in all its three elements, while a number can merely represent its magnitude.

41. Whatever number of forces may act upon one point of a body, and whatever their direction, they can impart to the body only one single motion in one certain direction.

42. When several forces (termed *components*) act on a point, tending to produce motion in different directions, they may be incorporated into one force, called the *resultant*, which, acting alone, will have the same mechanical effect as the several components.

43. When any number of forces act on a point in the same straight line, the resultant is equal to their sum, if they act in the same direction; but if they act in opposite directions, the resultant is equal to the difference between the sum of those acting in one direction and the sum of those acting in the other.

44. If two forces acting upon the same point be represented in magnitude and direction by two lines drawn through that point, then the resultant of such forces will be represented in magnitude and direction by the diagonal of the *parallelogram*, of which these lines are the sides.

45. If any number of forces, A, B, C, D, &c., act upon the same point in any direction whatever, and in any plane whatever, by first finding the resultant of A and B, then of this resultant and C, then of this resultant and D, and so on, we shall finally arrive at the determination of a single force, which will be mechanically equivalent to, and will therefore be the resultant of the entire system.

46. If the components act in the same plane, the resultant is found by means of what is technically termed the *parallelogram* of forces, if in different planes by the *parallelopiped* of forces.

47. The resultant of two forces which act on different points of the same body in parallel lines and in the same direction, is a single force equal to their sum, acting parallel to them, and in the same direction, at an intermediate point, which divides the line joining the two points of application of the components, in the inverse ratio of the magnitudes of these components.

48. The resultant of two forces, which act on different points of the same body in parallel lines but in opposite directions, is a single force equal to their difference, acting parallel to them and in the direction of the greater force, and at a point beyond the greater of the two forces, so situated, that the point of application of the greater of the two forces divides the distance between the points of application of the smaller force and of the resultant in the inverse ratio of the magnitudes of the smaller force and of the resultant.

49. When any number of parallel forces, A, B, C, D, &c., act on a body, at any point whatever, and in any planes whatever, by first finding the resultant of A and B, next of this resultant and C, then of this last resultant and D, and so on, we shall finally arrive at the determination of a single force, which will be mechanically equivalent to, and will therefore be the resultant of the entire system of parallel forces.

50. When a system of forces consists of two equal opposite and parallel forces, it is called a Couple.

51. Two equal and parallel forces acting on a body in contrary directions have a tendency to make that body revolve round an axis perpendicular to a plane passing through the direction of such two parallel and opposite forces; and such tendency is porportional to the product obtained by multiplying the magnitude of the forces by the distance between their points of application; and consequently, all couples, in which such products are equal, and which have their planes parallel, are mechanically equivalent, provided their tendency is to turn the body round in the same direction; but if two such couples have a tendency to turn the body in contrary directions then they have equal and contrary mechanical effects, and would, if simultaneously applied to the same body, keep it in equilibrium.

52. If any two forces, not parallel in direction, but which are in the same plane, be applied at any two points of a body, they admit of a single resultant, which may be determined by producing the lines that in magnitude and direction represent the two forces, until they meet in a point, and then applying the principle of the parallelogram of forces.

53. If two forces not parallel in direction act in different planes on two points of a body, they are mechanically equal to the combined action of a couple and of a single force, and their effect will be two-fold—1st, a tendency to produce revolution; 2nd, a tendency to produce progressive motion, so that, if not held in equilibrium by some antagonistic forces, the body will at the same time move forward, and revolve round some determinate axis.

54. The process of incorporating or compounding two or more forces into one, is called the composition of forces; that of separating or resolving a single force into two or more, is termed the resolution of forces.

55. As all the molecules of a body may be considered as gravitating in parallel lines towards the centre of the earth

—these parallel forces may (Art. 49) be compounded into a single force—which resultant is equal to the sum of all the forces affecting the particles severally; or in other words, to the weight of the mass. The point to which this resultant is applied, is called the *Centre of Gravity*, and the vertical line in which it acts is termed the *Line of Direction*.

56. Every dense body or solid mass possesses a centre of gravity.

NOTE.—The centre of gravity is sometimes called the *Centre of Inertia;* because, if it be moved, the whole mass is moved—it is likewise called the *Centre of Parallel Forces*, for the reason assigned in Art. 55.

57. The *Centre of Gravity* may be defined to be that point in a body, upon which, if the body be supported, it remains at rest and is balanced in any and every position.

58. If a body, regular or irregular in shape, be freely suspended by a point, the centre of gravity will invariably lie in the line of suspension. If suspended by several points of succession, the lines of suspension will have a common point of intersection, which point will be the centre of gravity of the body.

59. The *Centre of Gravity* is not necessarily *in* the body but may be in some adjoining space, as is the case in a ring, a table, an empty box, &c.

60. The tendency of a body, when free to move in any direction, is always to rest with the centre of gravity as low as possible.

61. The *Stability* of a body resting in any position is estimated by the magnitude of the force required to disturb or overturn it, and will therefore depend on the position of the centre of gravity with reference to the point of support.

62. A body supported on the centre of gravity is said to be in a condition of *Neutral or Indifferent Equilibrium*; when the point of support is *above* the centre of gravity the body is said to be in a condition of *Stable Equilibrium*, when the point of support is *beneath* the centre of gravity the body is said to be in a condition of *Unstable Equilibrium*.

63. The centre of gravity of two separate bodies may be found by dividing the line joining their centres in the inverse ratio of the magnitudes of the bodies.

CHAPTER III.

MECHANICAL POWERS.

64. The object of all Mechanical contrivances is

1st. To gain power at the expense of velocity; or
2nd. To gain velocity at the sacrifice of force.

65. The relative gain and loss of power and velocity is regulated by that principle in philosophy known as the Law of Virtual Velocities, or the Equality of Moments.

66. The law of Virtual Velocity may be thus enunciated:—

If in any machine the power and weight be in equilibrium, and the whole be put in motion, then the power multiplied by the units of distance through which it moves is equal to the weight multiplied by the units of distance through which it moves.

Or if P = *power,* W = *weight,* S = *space moved through by* P, *and* s = *space through which* W *moves.*

Then $P : W :: s : S$.

Hence $P = \frac{W \times s}{S}$; $S = \frac{W \times s}{P}$, $W = \frac{P \times S}{s}$ and $s = \frac{P \times S}{W}$

EXAMPLE 5.—A weight of 700 lbs. is moved through 90 feet by a certain power moving through 5,100 feet. Required the power.

SOLUTION.

Here $W = 700$, $s = 90$ and $S = 5100$.

Hence $P = \frac{W \times s}{S} = \frac{700 \times 90}{5100} = 12\frac{6}{17}$ lbs. *Ans.*

EXAMPLE 6.—A weight of 500 lbs. is moved by a power of 20 lbs.; through how many feet must the power move in order to raise the weight through 16 feet?

SOLUTION.

Here $W = 500$, $P = 20$ and $s = 16$.

Hence $S = \frac{W \times s}{P} = \frac{500 \times 16}{20} = 400$ feet. *Ans.*

EXAMPLE 7.—A power of 21 lbs. moving through 75 feet carries a certain weight through 11 feet. Required the weight.

Here $P = 21$, $S = 75$ and $s = 11$.

Then $W = \frac{P \times S}{s} = \frac{21 \times 75}{11} = 143\frac{2}{11}$ lbs. *Ans.*

EXAMPLE 8.—A power of 204 lbs. moving through 30 feet is made to move a weight of 1000 lbs. Through how many feet does the weight move?

SOLUTION.

Here $P = 204$, $W = 1000$, and $S = 30$.

Then $s = \frac{P + S}{W} = \frac{204 \times 30}{1000} = 6\frac{3}{25}$ ft. *Ans.*

EXERCISE.

9. A power of 7 lbs. is made to move a weight of 1000 lbs. through 11 feet; through how many feet must the power move? *Ans.* $1571\frac{3}{7}$ feet.

10. A power of 97 lbs. moving through 86 feet raises a certain weight through ten feet. Required the weight. *Ans.* $834\frac{1}{5}$ lbs.

11. A weight of 888 lbs. is raised by a power of 60 lbs; through how many feet must the power move in order to raise the weight through 1 foot? *Ans.* $14\frac{4}{5}$ feet.

12. A certain power moving through 27 feet is so applied that it carries a weight of 2500 lbs. through 4 feet. Required the power. *Ans.* $370\frac{10}{27}$ lbs.

---

67. Any contrivance by which, in accordance with the principle of Virtual Velocities, a small force acting through a large space is converted into a great force acting through a small space, or *vice versâ,* is a machine. Machines are either simple or complex.

68. In the composition of machinery it is usual to speak of six mechanical powers—more properly termed Mechanical elements, or Simple Machines, viz:—

The Lever,
The Inclined Plane,
The Pulley and Cord, } *Primary Mechanical Elements.*

The Wheel and Axle,
The Wedge,
The Screw, } *Secondary Mechanical Elements.*

69. In reality, however, there are but two simple mechanical elements, viz: the Lever and the Inclined Plane. The Wheel and Axle and the Pulley are merely modifications of the *lever*, while the Wedge and the Screw are both formed from the *inclined plane*.

70. In theoretical mechanics levers are assumed to be perfectly *rigid and imponderable*—cords, ropes and chains are regarded as having neither thickness, stiffness nor weight, they are assumed to be mere *mathematical lines, infinitely flexible and infinitely strong.* At first no allowance is made for friction, atmospheric resistance, &c. After the problem, divested of all these complicating circumstances, has been solved, the result is modified by taking into consideration the effects of weight, friction, atmospheric resistance, rigidity of cords, flexibility of bars, &c.

THE LEVER.

71. The lever is a bar of wood, or iron, movable about a fixed point or pivot called the Fulcrum.

72. Levers are either Straight or Bent, Simple or Compound.

73. Of simple Straight Levers there are three kinds,—the distinction depending upon the relative positions of the fulcrum, the power, and the weight.

74. In levers of the first class the *fulcrum* is between the *power* and the *weight.*

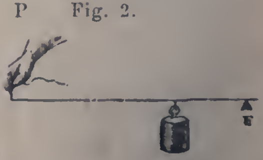
Fig. 1.

Of this kind of lever, we may mention as examples, a pair of scissors, pliers, or pincers, a pump-handle, the beam of a pair of scales, a crowbar when used for prying, &c.

75. In levers of the second class the *weight* is between the *fulcrum* and the *power.*

Fig. 2.

Nutcrackers, an oar in rowing, a crowbar when used in lifting, &c., are examples of levers of the second kind.



76. In levers of the third class the *power* is between *the fulcrum* and the *weight.*

Fig. 3.

A pair of common tongs, sheep-shears, the treadle of a foot lathe, a door when opened or closed by placing the hand near the hinge, afford examples of levers of the third class.

NOTE.—In levers of the first class the power may be either greater or less than the weight; in levers of the second class, the power is *always less* than the weight; and in levers of the third class, the power is *always greater* than the weight. Hence levers of the third class are called *losing levers*, and are used merely to secure extent of motion. Most of the levers in the animal economy are levers of the third kind.

77. That portion of the lever included between the fulcrum and the weight is termed the *arm of the weight:* that portion between the fulcrum and the power is termed the *arm of the power.*

The power and the weight in the lever are in equilibrium when the power is to the weight as the arm of the weight is to the arm of the power.

Or let P = *power,* W = *the weight,* A = *the arm of the power, and* a = *the arm of the weight.*

Then $P : W :: a : A$.

Hence $P = \frac{W \times a}{A}$; $W = \frac{P \times A}{a}$; $a = \frac{P \times A}{W}$; and $A = \frac{W \times a}{P}$

EXAMPLE 13.—The power-arm of a lever is 11 feet long, the arm of the weight 3 feet long, the weight is 93 lbs. Required the power.

SOLUTION.

Here $W = 93$, $A = 11$ and $a = 3$.

Then $P = \frac{W \times a}{A} = \frac{93 \times 3}{11} = 25\frac{4}{11}$ lbs. *Ans.*

EXAMPLE 14.—The power-arm of a lever is 17 feet long, the arm of the weight is 20 feet long, the power is 110 lbs. What is the weight?

SOLUTION.

Here $P = 110$ lbs. $A = 17$, and $a = 20$.

Then $W = \frac{P \times A}{a} = \frac{110 \times 17}{20} = 93\frac{1}{2}$ lbs. *Ans.*

EXAMPLE 15.—By means of a lever a power of 4 oz. is made to balance a weight of 7 lbs. Avoir.; the arm of the weight is $2\frac{1}{2}$ inches long. Required the arm of the power.

SOLUTION.

Here $P = 4$ oz., $W = 7$ lbs., $= 112$ oz., and $a = 2\frac{1}{2}$.

Then $A = \dfrac{W \times a}{P} = \dfrac{112 \times 2\frac{1}{2}}{4} = 70$ inches. *Ans.*

EXERCISE.

16. The power-arm of a lever is 16 feet long, the arm of the weight 2 feet long, and the weight is 250 lbs. Required the power. *Ans.* $31\frac{1}{4}$ lbs.
17. The power-arm of a lever is 20 feet long, the arm of the weight 70 feet; what power will balance a weight of 5 cwt.? *Ans.* $17\frac{1}{2}$ cwt.
18. The power-arm of a lever is 60 inches long, the arm of the weight 90 inches long, the power is 76 lbs. Required the weight. *Ans.* $50\frac{2}{3}$ lbs.
19. The power-arm of a lever is 17 feet long, the arm of the weight 19 feet; what power will balance a weight of 950 lbs.? *Ans.* $1061\frac{13}{17}$ lbs.
20. The power-arm of a lever is 12 feet long, the power is 10 lbs., and the weight 75 lbs. Required the length of the arm of the weight. *Ans.* $1\frac{3}{5}$ feet.
21. By means of a lever a power of $12\frac{3}{4}$ lbs. is made to balance a weight of 93 lbs.; the arm of the weight being $6\frac{1}{2}$ feet, what is the length of the arm of the power? *Ans.* $47\frac{7}{17}$ ft.

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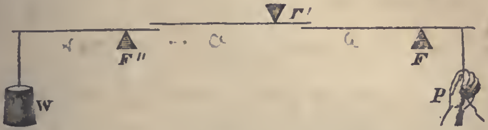
78. When the power and the weight merely balance each other, i. e., when no motion is produced, there is no difference between the second and third classes of levers since neither force can be regarded as the mover or the moved. In order to produce motion, one of these forces must prevail, and the lever then belongs to the second or third class, according as the force *nearer to* or *further from* the fulcrum prevails.

79. If the arms of the lever are curved or bent, their effective lengths must be ascertained by perpendiculars drawn from the fulcrum upon the lines of direction of the power and the weight; the same rule must be adopted when the lever is straight, if the power and weight do not act parallel with one another.

THE COMPOUND LEVER.

80. Two or more simple levers acting upon one another constitute what is called a Compound Lever or Com-

Fig. 4.



position of Levers. In such a combination the ratio of the power to the weight is compounded of the ratios existing between the several arms of the compound lever.

81. *In the compound lever if W = weight, P = power, $a\ a'\ a''$ the arms of the weight, and $A\ A'\ A''$ the arms of the power.*

Then $P : W :: a \times a' \times a'' : A \times A' \times A''$

Hence $P = \dfrac{W \times a \times a' \times a''}{A \times A' \times A''}$ and $W = \dfrac{P \times A \times A' \times A''}{a \times a' \times a''}$

EXAMPLE 22.—In a combination of levers the arms of the power are 6, 7 and 11 feet, the arms of the weight 2, 3, and $3\frac{1}{2}$ feet, the weight is 803 lbs.; what is the power?

SOLUTION.

Here $W = 803$ lbs., $a = 2$, $a' = 3$, $a'' = 3\frac{1}{2}$, $A = 6$, $A' = 7$, $A'' = 11$.

Then $P = \dfrac{W \times a \times a' \times a''}{A \times A' \times A''} = \dfrac{803 \times 2 \times 3 \times 3\frac{1}{2}}{6 \times 7 11} = 36\frac{1}{2}$ lbs. *Ans.*

EXAMPLE 23.—In a compound lever the power is 17 lbs., the arms of the power 9, 7, 6, 5, and 4 ft., and the arms of the weight 2, 3, 1, 1, and $\frac{1}{2}$ ft. Required the weight.

SOLUTION.

Here $P = 17$ lbs. $A = 9$, $A' = 7$, $A'' = 6$, $A''' = 5$, $A'''' = 4$, $a = 2$, $a' = 3$, $a'' = 1$, $a''' = 1$, and $a'''' = \frac{1}{2}$.

Then $W = \dfrac{P \times A \times A' \times A'' \times A''' \times A''''}{a \times a' \times a'' \times a''' \times a''''} = \dfrac{17 \times 9 \times 7 \times 6 \times 5 \times 4}{2 \times 3 \times 1 \times 1 \times \frac{1}{2}} = \dfrac{128520}{2}$
$= 64260$ lbs. *Ans.*

EXERCISE.

24. In a compound lever the arms of the power are 9 and 17 ft., the arms of the weight 3 and 4 ft., the power is 19 lbs. What is the weight? *Ans.* $242\frac{1}{4}$ lbs

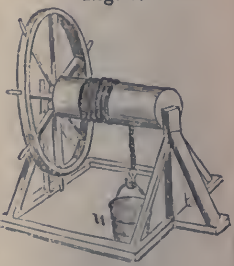
25. In a compound lever the arms of the power are 6, 8, 10, and 12 ft., the arms of the weight, 7, 5, 3, and 1 ft., the weight is 700 lbs. Required the power. *Ans.* $12\frac{73}{96}$.

26. In a compound lever the arms of the weight are 11, 13, and 9 ft., the arms of the power are, 4, 7, and 2 ft., the weight is 560 lbs. What is the power? *Ans.* 12870 lbs.

THE WHEEL AND AXLE.

82. The wheel and axle consists of a wheel with a cylindrical axle passing through its centre, perpendicular to the plane of the wheel. The power is applied to the circumference of the wheel, and the weight to the circumference of the axle.

Fig. 5.



83. The wheel and axle is merely a modification of the lever with unequal arms; the radius of the wheel corresponding to the arm of the power, and the radius of the axle to the arm of the weight.

84. The wheel and axle is sometimes called the *continual* or *perpetual lever*, because the power acts continually on the weight.

85. *The power and weight in the wheel and axle are in equilibrium when the power is to the weight as the radius of the axle is to the radius of the wheel.*

86. *For the wheel and axle—let* P = *the power*, W = *the weight*, r = *radius of the axle*, R = *radius of the wheel.*

Then $P : W :: r : R$.

Hence $P = \frac{W \times r}{R}$; $W = \frac{P \times R}{r}$; $r = \frac{P \times R}{W}$; and $R = \frac{W \times r}{P}$

EXAMPLE 27.—In a wheel and axle the radius of the axle is 7 inches, the radius of the wheel is 35 inches. What power will balance a weight of 643 lbs?

SOLUTION.

Here $W = 643$ lbs., $R = 35$ inches, and $r = 7$ inches.

Then $P = \frac{W \times r}{R} = \frac{643 \times 7}{35} = 128\frac{3}{5}$. *Ans.*

EXAMPLE 28.—In a wheel and axle the radius of the axle is 6 inches, the radius of the wheel is 27 inches. What weight will be balanced by a power of 123 lbs.?

SOLUTION.

Here $P = 123$ lbs., $R = 27$ in., and $r = 6$ in.

Then $W = \frac{P \times R}{r} = \frac{123 \times 27}{6} = 553\frac{1}{2}$ lbs. *Ans.*

EXAMPLE 29.—By means of a wheel and axle a power of 11 lbs. is made to balance a weight of 719 lbs., the radius of the axle is 3 inches. Required the radius of the wheel.

SOLUTION.

Here $W = 719$ lbs., $P = 11$ lbs., and $r = 3$ in.

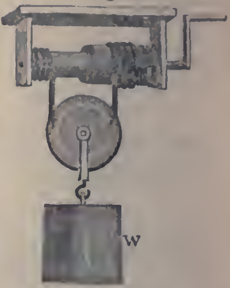
Then $R = \frac{W \times r}{P} = \frac{719 \times 3}{11} = 196\frac{1}{11}$ inches. *Ans.*

EXERCISE.

30. In a wheel and axle the radius of the axle is 7 inches, the radius of the wheel is 70 inches. What power will balance a weight of 917 lbs.? *Ans.* $91\frac{7}{10}$ lbs.
31. In a wheel and axle the radius of the axle is 5 inches, and the radius of the wheel 17 inches. What power will balance a weight of 5950 lbs.? *Ans* 1750 lbs.
32. In a wheel and axle the radius of the axle is 9 inches, and the radius of the wheel is 37 inches. What power will balance a weight of 925 lbs.? *Ans.* 225 lbs.
33. In a wheel and axle the radius of the axle is 11 inches, and the radius of the wheel is 45 inches. What weight will a power of 17 lbs. balance? *Ans.* $69\frac{6}{11}$ lbs.
34. By means of a wheel and axle a power of 37 lbs. balances a weight of 700 lbs., the radius of the axle being 8 inches, what is the radius of the wheel? *Ans.* $151\frac{13}{37}$ inches.
35. By means of a wheel and axle a power of 22 lbs. balances a weight of 870 lbs. If the radius of the wheel be 67 inches, what will be the radius of the axle? *Ans.* $1\frac{302}{435}$ inches.

THE DIFFERENTIAL WHEEL AND AXLE.

Fig. 6.



87. In the differential wheel and axle, the axle consists of two parts, one thicker than the other. By each revolution of the wheel the rope rolls once off the thinner portion and once on the thicker portion, and is consequently shortened only by the differences between the circumferences of the axles; and the distance through which the weight is raised is equal to *half the shortening of the rope.* The effect is therefore the same as if an axle had been used with a radius equal to half the difference between the radii of the thicker and thinner parts of the differential axle.*

88. *For the differential wheel and axle let* d = *the difference between the radii of the axles,* R = *radius of the wheel,* P = *the power, and* W = *the weight.*

Then $P : W :: \frac{1}{2}d : R$.

Whence $P = \frac{W \times \frac{1}{2}d}{R}$, $W = \frac{P \times R}{\frac{1}{2}d}$, $R = \frac{W \times \frac{1}{2}d}{P}$, and $d = \frac{P \times R}{\frac{1}{2}W}$

EXAMPLE 36.—In a differential wheel and axle the radius of the larger axle is $4\frac{1}{5}$ inches, the radius of the smaller axle is $4\frac{1}{6}$ inches, the radius of the wheel is 70 inches. What power will balance a weight of 1000 lbs.?

SOLUTION.

Here d = difference of radii $= \frac{1}{5} - \frac{1}{6}, = \frac{1}{30}$, $W = 1000$ lbs., $R = 70$ in.

Then $P = \frac{W \times \frac{1}{2}d}{R} = \frac{1000 \times \frac{1}{60}}{70} = \frac{\frac{1000}{60}}{\frac{70}{1}} = \frac{1000}{4200} = \frac{5}{21}$ lbs. *Ans.*

EXAMPLE 37.—In a differential wheel and axle the radii of the axles are $2\frac{1}{7}$ and $2\frac{3}{20}$ inches, the radius of the wheel is 100 inches. What power will balance a weight of 7234 lbs.?

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* The radii being proportional to the circumferences.

SOLUTION.

Here $d = \frac{1}{7} - \frac{3}{29} = \frac{8}{203}$ in. $R = 100$, and $W = 7234$.

Then $P = \frac{W \times \frac{1}{2}d}{R} = \frac{7234 \times \frac{4}{203}}{100} = 1\frac{2159}{5075}$ lbs. *Ans.*

EXAMPLE 38.—In a differential wheel and axle the radii of the axles are $3\frac{1}{8}$ and $3\frac{2}{17}$ inches, the radius of the wheel is 86 inches. What weight will a power of 17 lbs. balance ?

SOLUTION.

Here $d = \frac{1}{8} - \frac{2}{17} = \frac{1}{136}$ of an inch, $R = 86$ inches, and $P = 17$ lbs.

Then $W = \frac{P \times R}{\frac{1}{2}d} = \frac{17 \times 86}{\frac{1}{272}} = \frac{1462}{\frac{1}{272}} = 397664$ lbs. *Ans.*

EXAMPLE 39.—In a differential wheel and axle the radius of the wheel is 32 inches, and a power of 5 lbs. balances a weight of 729. What is the difference between the radii of the axles ?

SOLUTION.

Here $W = 729$ lbs., $P = 5$ lbs., and $R = 32$ inches.

Then $d = \frac{P \times R}{\frac{1}{2}W} = \frac{5 \times 32}{\frac{1}{2} \text{ of } 729} = \frac{160}{\frac{729}{2}} = \frac{320}{729}$ of an inch. *Ans.*

EXERCISE.

40. In a differential wheel and axle the radii of the axles are $7\frac{1}{9}$ and $7\frac{2}{21}$ inches, and the radius of the wheel is 85 inches. What power will balance a weight of 6900 lbs. ?
Ans. $\frac{230}{357}$ lbs ?

41. In a differential wheel and axie, the radii of the axles are 17 and 16 inches, and the radius of the wheel is 130 inches. What weight will a power of 17 lbs. balance ?
Ans. 4420 lbs.

42. In a differential wheel and axle, the radii of the axles are $2\frac{1}{3}$ and $2\frac{2}{7}$ inches, and a power of $23\frac{1}{2}$ oz. balances a weight of 6400 oz. Required the radius of the wheel.
Ans. $6\frac{478}{987}$ inches.

43. In a differential wheel and axle, the radii of the axles are $4\frac{1}{5}$ and 5 inches, the radius of the wheel being 120 inches. What power will balance a weight of 2430 oz. ?
Ans. $8\frac{1}{10}$ oz.

44. In a differential wheel and axle, the radii of the axles are $1\frac{3}{8}$ and $1\frac{2}{9}$ feet, the radius of the wheel is $12\frac{3}{4}$ feet. What weight will a power of 880 lbs. balance ? *Ans.* 146880 lbs.

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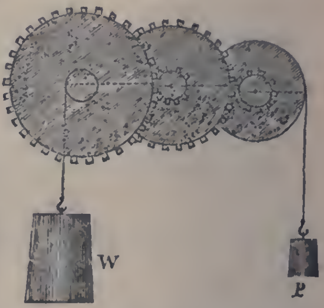
89. Since the wheel and axle is merely a modification of the lever, a system of wheels and axles is simply a modification of the compound lever, and the conditions of

equilibrium are the same, i. e., the ratio of the power to the weight is compounded of the ratios of the radii of the axles to the radii of the wheels. In *toothed gear*, however, owing to the difficulty in determining the effective radii of wheel and axle, the ratio of the power to the weight is determined by the number of teeth and leaves upon the wheel and pinion.

90. Axles are made to act on wheels by various methods —as by the mere friction of their surfaces, by straps or endless bands, &c.; but the most common method of transmitting motion through a train of wheelwork is by means of teeth or cogs raised upon the circumferences of the wheels and axles.

91. When cogged wheels and axles are employed, that part of the axle bearing the cogs is called a *pinion*. The cogs raised upon the pinion are called *leaves*, those upon the wheel are termed *teeth*.

Fig. 7.



92. Wheelwork may be used either to concentrate or diffuse power. The power is concentrated when the pinions turn the wheels, as is the case in the *crane* which is used to gain power. The power is diffused when the wheels turn the pinions, as is the case in the fanning mill, threshing machine, &c., where extent of motion is sought.

93. *In a system of toothed wheels and pinions, the conditions of equilibrium are,—that the power is to the weight as the continued product of all the leaves is to the continued product of all the teeth.*

94. *For a train of wheel work let* $P =$ *the power,* $W =$ *the weight* $t\ t'\ t'' =$ *the teeth of the wheel, and* $l\ l'\ l'' =$ *the leaves of the pinion.*

Then $P : W :: l \times l' \times l'' : t \times t' \times t''$.

Hence $P = \frac{W \times l \times l' \times l''}{t \times t' \times t''}$, and $W = \frac{P \times t \times t' \times t''}{l \times l' \times l''}$

EXAMPLE 45.—The number of teeth in each of three successive wheels is 80 and the number of leaves in each of the pinions is 5. With this machine what weight will be supported by a power of 17 lbs?

SOLUTION.

Here $P = 17$, $t = 80$, $t' = 80$, $t'' = 80$, $l = 5$, $l' = 5$ and $l'' = 5$.

Then $W = \frac{P \times t \times t' \times t''}{l \times l' \times l''} = \frac{17 \times 80 \times 80 \times 80}{5 \times 5 \times 5} = \frac{8704000}{125} = 69632$ lbs. *Ans.*

EXAMPLE 46.—In a train of wheel work there are four wheels and four axles, the first wheel and the fourth axle plain, (i. e. without cogs), and having radii respectively of 10 and 2 feet. The second wheel has 60, the third 90 and the fourth 70 teeth, the first axle 7, the second 5 and the third 9 leaves. What power will hold in equilibrium a weight of 20000 lbs?

SOLUTION.

Here we have a combination of the simple wheel and axle and a system of cogged wheels and axles.

$W = 20000$ lbs. $R = 10$, $r = 2$, $t = 60$, $t' = 90$, $t'' = 70$, $l = 7$, $l' = 5$ and $l'' = 9$.

Then cogged wheels and axles acting alone, $P = \frac{20000 \times 7 \times 5 \times 9}{60 \times 90 \times 70} = 16\frac{2}{3}$ lbs.

and so far as the action of the plain wheel and axles is concerned this $16\frac{2}{3}$ lbs. becomes the weight.

Then $P = \frac{W \times r}{R} = \frac{16\frac{2}{3} \times 2}{10} = \frac{33\frac{1}{3}}{10} = 3\frac{1}{3}$ lbs. *Ans.*

EXAMPLE 47.—In a train of wheel work there are three wheels and axles, the first wheel and the last axle plain, and having a radius of 9 and 3 feet respectively—the cogged wheels have respectively 80 and 110 teeth, and the pinions 11 and 8 leaves What weight will a power of 100 lbs. sustain?

SOLUTION.

Here $P = 100$ lbs., $R = 9$, $r = 3$, $t = 80$, $t' = 110$, $l = 11$, and $l' = 8$

Then for cogged wheel work acting alone, $W = \frac{P \times t \times t'}{l \times l'} = \frac{100 \times 80 \times 110}{11 \times 8}$
$= \frac{880000}{88} = 10000$ lbs.

For plain wheel and axle alone, $W = \frac{P \times R}{r} = \frac{10000 \times 9}{3} = \frac{90000}{3}$
$= 30000$ lbs. *Ans.*

EXERCISE.

48. In a system of wheel work there are five wheels and pinions. The wheels have respectively 100, 90, 80, 70 and 60 teeth, and the pinions respectively 9, 7, 11, 9 and 7 leaves; with such an appliance, what weight would be sustained by a power of 77 lbs.? *Ans.* $533333\frac{1}{3}$ lbs.

49. In a train of four wheels and axles the wheels have respectively 70, 65, 60 and 50 teeth, and the axles respectively 9, 8, 7 and 6 leaves; with such an instrument, what power could support a weight of 13000 lbs? *Ans.* $2\frac{22}{25}$ lbs.

50. In a train of wheel work there are three wheels and three axles, the first wheel and last axle plain, and having radii respectively 6 and 2 feet. The second and third wheels have respectively 80 and 50 teeth, and the first and second pinions respectively 5 and 8 leaves. With such a machine what weight will be balanced by a power of 11 lbs? *Ans.* 3300 lbs.

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95. In ordinary wheel work it is usual, in any wheel and pinion that act on each other, to use numbers of teeth that are prime to each other so that each tooth of the pinion may encounter every tooth of the wheel in succession, that thus if any irregularities exist, they may tend to diminish one another by constant wear. This odd tooth in the wheel is termed the *hunting cog*.

Thus if a pinion contain 10 leaves and the wheel 101 teeth, it is evident that the wheel must turn round 101 times and the pinion 10×101 or 1010 times before the same leaves and the teeth will be again engaged.

96. Wheels are divided into *crown*, *spur*, and *bevelled gear*.

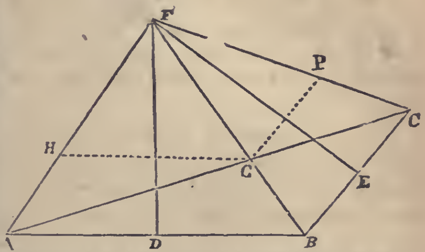
97. The *crown wheel* has its teeth perpendicular to its plane; the *spur wheel* has its teeth, which are continuations of its radii, placed on its rim; the *bevelled wheel* has its teeth obliquely placed, i. e., raised on a surface inclined at any angle to the plane of the wheel.

98. To communicate motion round parallel axes spur-gear is employed: bevelled gear is used when the axes of motion are inclined to one another at any proposed angle. Where the axes are at right angles to one another a crown wheel working in a spur pinion or a crown pinion working in a spur wheel is usually employed.

99. Bevelled wheels are always frusta of cones channelled from their apices to their bases.

Note.—When bevelled wheels of different diameters are to work together the sections of the cones of which they are to be frusta are found in the following manner: Let A B be the diameter of the large wheel, and B C that of the smaller. Place A B and B C so as to include the proposed angle. Bisect A B in D and B C in E. Draw perpendiculars D F, E. F meeting in F and join F A, F B and F C. Then F A B and F B C are sections of the required cones. Also drawing H G parallel to A B, and G P parallel to B C, we obtain H A B G, and G B C P any required frusta.

Fig. 8.



THE PULLEY.

100. The Pulley is a circular disc of wood or iron, grooved on the edge and made to turn on its axis by means of a cord or rope passing over it.

101. The pulley is merely a modification of the lever with equal arms, and hence no mechanical advantage is gained by using it—the theory of its use being just as perfect if the cord be passed through rings or over perfectly smooth surfaces. The real advantage of the pulley and cord as a mechanical power is due to the equal tensions of every part of the cord, i. e., is founded upon the fact that the same flexible cord, free to run over pulleys or through smooth rings in every direction, must always undergo the same amount of tension in every part of its length.

102. The pulley is called either fixed or movable according as its *axis* is fixed or movable.

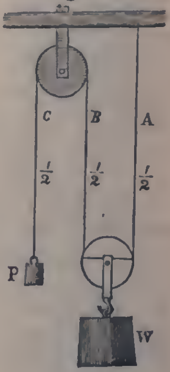
103. Movable pulleys are used either singly, in which case they are called *runners*, or in combination. Systems of pulleys are worked either by one cord or by several cords. Pulleys worked by more than one cord are called *Spanish Bartons.*

104. The pulley is often called a *sheaf*, and the case in which it turns a *block*. A block may contain many sheaves. A combination of ropes, blocks, and sheaves, is called a *tackle*.

105. In the single fixed pulley the power must be equal to the weight, i. e., a fixed pulley does not concentrate force at all. And hence the only mechanical advantage derived from its use is, that it changes the direction of the power.

106. *In a system of pulleys moved by one cord the conditions of equilibrium are that the power is to the weight as* 1 *is to twice the number of movable pulleys.*

Fig. 9.



This is evident from the fact that the weight is sustained equally by every part of the cord, and, neglecting the last fold or that to which the power is attached, there are two folds of cord for every movable pulley. Thus in Fig. 9 the weight is sustained by A and B, each bearing ½ of it; and since B passes over a fixed pulley, the power attached to C must be equal to the tension exerted on B = ½ the weight.

107. *For a system of pulleys moved by one cord let* P = *the power*, W = *the weight, and* n = *the number of movable pulleys.*

Then $P : W :: 1 : 2n$.

$$\text{Hence } P = \frac{W}{2n},\ W = P \times 2n,\ n = \frac{W}{2P}.$$

EXAMPLE 51.—In a system of pulleys worked by a single cord there are 4 movable pulleys. What power will support a weight of 804 lbs?

SOLUTION.

Here $W = 804$ and $n = 4$

$$\text{Hence } P = \frac{W}{2 \times n} = \frac{804}{2 \times 4} = \frac{804}{8} = 100\tfrac{1}{2} \text{ lbs. } Ans.$$

EXAMPLE 52.—In a system of 7 movable pulleys worked by a single cord, what weight will be supported by a power of 17 lbs.?

SOLUTION.

Here $P = 17$ and $n = 7$.

Hence $W = P \times 2 \times n = 17 \times 2 \times 7 = 17 \times 14 = 238$ lbs. *Ans.*

EXAMPLE 53.—In a system of movable pulleys worked by a single cord a power of 7 lbs. balances a weight of 84 lbs.; how many movable pulleys are there in the combination?

SOLUTION.

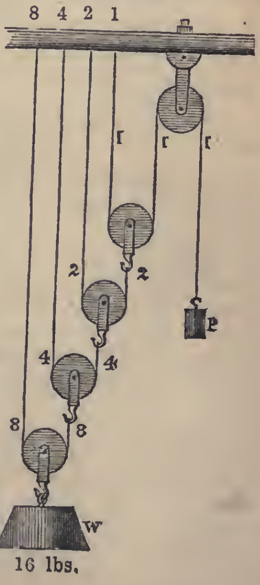
Here $P=7$ lbs. and $W=84$ lbs.

Hence $n=\frac{W}{2\times P}=\frac{84}{2\times 7}=\frac{84}{14}=6$. *Ans.*

EXERCISE.

54. In a system of six movable pulleys worked by one cord the weight is 700 lbs. What is the power? *Ans.* $58\frac{1}{3}$. lbs.
55. In a system of eleven movable pulleys worked by one cord the weight is 6325 lbs. Required the power. *Ans.* $116\frac{1}{2}$ lbs.
56. In a system of eight movable pulleys worked by one cord the power is 37 lbs. Required the weight. *Ans.* 592 lbs.
57. In a system of seven movable pulleys worked by a single cord the power is 13 lbs.; what is the weight? *Ans.* 182 lbs.
58. In a system of movable pulleys worked by a single cord, a power of 35 lbs. supports a weight of 7000 lbs. How many movable pulleys are there in the combination? *Ans.* 100.

Fig. 10.



108. In a system of pulleys, such as represented in figure 10, where each movable pulley hangs by a separate cord, one extremity of each cord being attached to a movable pulley and the other to a hook in a beam or other fixed support, each pulley doubles the effect, and the conditions of equilibrium are that *the power is to the weight as 1 is to 2 raised to the power indicated by the number of movable pulleys.*

NOTE.—This will become evident by attentively examining the diagram and following up the several cords. The figures at the top show the portion of weight borne by the several parts of the beam, those attached to the cords show the portion of the weight sustained by each part of the cord.

109. *For a system of pulleys, such as exemplified in Fig. 10, let* P = *the power,* W = *the weight, and* n = *the number of movable pulleys.*

Then P : W :: 1 : 2^n. *Hence* $P = \frac{W}{2^n}$ *and* $W = P \times 2^n$.

EXAMPLE 59.—In a system of pulleys of the form indicated in Fig. 10, there are 5 movable pulleys, and a weight of 128 lbs. What is the power?

SOLUTION.

Here $W = 128$ lbs. and $n = 5$.

Then $P = \frac{W}{2^n} = \frac{128}{2^5} = \frac{128}{32} = 4$ lbs. *Ans.*

EXAMPLE 60.—In such a system of pulleys as is shewn Fig. 10, there are 7 movable pulleys. What weight will a power of 11 lbs. balance?

SOLUTION.

Here $P = 11$ and $n = 7$.
Hence $W = P \times 2^n = 11 \times 2^7 = 11 \times 123 = 1408$ lbs. *Ans.*

EXERCISE.

61. In the system of pulleys represented in Fig. 10, where there are 6 movable pulleys, what power will sustain a weight of 8000 lbs.? *Ans.* 125 lbs.

62. In such a system when there are 10 movable pulleys, what power will sustain a weight of 48000 lbs.? *Ans.* $46\frac{7}{8}$ lbs.

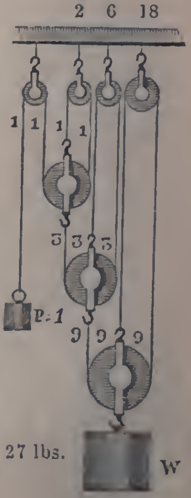
63. In such a system when there are 7 movable pulleys, what power will support a weight of 4564 lbs. *Ans.* $35\frac{21}{32}$ lbs.

64. In such a system when there are 3 movable pulleys, what weight will be sustained by a power of 17 lbs. *Ans.* 136 lbs.

65. In such a system what weight will a power of 70 lbs. support when there are 5 movable pulleys? *Ans.* 2240 lbs.

66. In such a system what weight will a power of 100 lbs. support when there are 11 movable pulleys? *Ans.* 204800 lbs.

Fig. 11.



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110. In a system of pulleys such as represented in Fig. 11 where the cord passes over a fixed pulley attached to the beam instead of being fastened to a hook in the beam, each movable pulley *triples* the effect, and the conditions of equilibrium are that *the power is to the weight as* 1 *to* 3 *raised to the power indicated by the number of movable pulleys.*

This will appear plain by a reference to the accompanying diagram where the numbers represent the same as in Art. 108.

111. *In a system such as is represented in Fig.* 11, *let* P = *power*, W = *the weight, and* n = *the number of movable pulleys.*
Then P : W :: 1 : 3^n.

Hence $P = \frac{W}{3^n}$ *and* $W = P \times 3^n$.

EXAMPLE 67.—In the system of pulleys represented in Fig. 11, what power will balance a weight of 4500 lbs., when there are 4 movable pulleys?

SOLUTION.

Here $W = 4500$ and $n = 4$.

Then $P = \frac{W}{3^n} = \frac{4500}{3^4} = \frac{4500}{81} = 55\frac{5}{9}$ lbs. *Ans.*

EXAMPLE 68.—In such a system when there are 6 movable pulleys, what weight will a power of 10 lbs. support?

SOLUTION.

Here $P = 10$, and $n = 6$.

Then $W = P \times 3^n = 10 \times 3^6 = 10 \times 729 = 7290$ lbs. *Ans.*

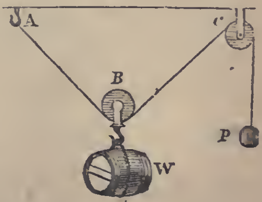
EXERCISE.

69. In the system of pulleys represented in figure 11, there are movable pulleys; what weight may be supported by a power of 10 lbs.? *Ans.* 2430 lbs.

70. In such a system there are 7 movable pulleys and the weight is 24057 lbs. Required the power. *Ans.* 11 lbs.

71. In such a system there are 9 movable pulleys — through how many feet must the power descend in order to raise the weight 10 feet? *Ans.* 196830 feet.

Fig. 12.



112. If the lines of direction of the power and weight make with one another an angle greater than 120°, the power will require to be greater than the weight; and as this angle approaches 180°, the difference between the power and weight will approach ∞. Hence it is impossible for any power P, however great, applied at P, to pull the cord ABC mathematically straight, and that however small the weight W may be.

THE INCLINED PLANE.

113. The Inclined Plane is regarded in mechanical science as a *perfectly hard, smooth, inflexible plane*, inclined obliquely to the weight or resistance.

114. There are two ways of indicating the degree of inclination of the inclined plane:

1st. By saying it rises so many feet, inches, &c., in a certain distance.

2nd. By describing it as rising at some stated angle with the horizon.

115. In the inclined plane the power may be applied in any one of three directions:

1st. Parallel to the plane.
2nd. Parallel to the base.
3rd. Inclined at any angle to the base.

116. In the inclined plane the conditions of equilibrium are as follows:—

1*st. If the power act parallel to the plane:—the power is to the weight as the height of the plane is to its length.*

2*nd. If the power act parallel to the base:—the power is to the weight as the height of the plane is to its base.*

NOTE.—The third case does not come within the design of the present work.

117. *For the inclined plane let* P = *the power*, W = *the weight*, L = *length of the plane*, H = *height of the plane*, *and* B = *base of the plane.*

Then $P : W :: H : L$.

Hence $P = \frac{W \times H}{L}$ $W = \frac{P \times L}{H}$; $H = \frac{P \times L}{W}$; *and* $L = \frac{W \times H}{P}$.

Also $P : W :: H : B$.

Hence $P = \frac{W \times H}{B}$; $W = \frac{P \times B}{H}$; $H = \frac{P \times B}{W}$; *and* $B = \frac{W \times H}{P}$.

EXAMPLE 72.—On an inclined plane rising 7 feet in 200, what power acting parallel with the plane will sustain a weight of 4000 lbs.?

SOLUTION.

Here $W = 4000$ lbs., $L = 200$, and $H = 7$.

Then $P = \frac{W \times H}{L} = \frac{4000 \times 7}{200} = \frac{28000}{200} = 140$ lbs. *Ans.*

EXAMPLE 73.—On an inclined plane rising 9 feet in 170—what weight will support a power of 180 lbs. acting parallel to the plane?

SOLUTION.

Here $P = 180$ lbs., $L = 170$, and $H = 9$.

Then $W = \frac{P \times L}{H} = \frac{180 \times 170}{9} = 3400$ lbs. *Ans.*

EXAMPLE 74.—On an inclined plane a power of 11 lbs. acting parallel to the plane supports a weight of 150 lbs.—how much does the plane rise in 200 feet?

SOLUTION.

Here $P = 11$ lbs., $W = 150$ lbs., $L = 200$ feet.

Then $H = \frac{P \times L}{W} = \frac{11 \times 200}{150} = 14$ feet 8 inches. *Ans.*

EXAMPLE 75.—The base of an inclined plane is 40 feet and the height 3 feet,—what power acting parallel to the base will support a weight of 250 lbs.?

SOLUTION.

Here $W = 250$ lbs., $H = 3$, and $B = 40$.

Then $P = \frac{W \times H}{B} = \frac{250 \times 3}{40} = 18\frac{3}{4}$ lbs. *Ans.*

EXAMPLE 76.—On an inclined plane a power of 9 lbs. acting parallel to the base supports a weight of 700 lbs.—the height of the plane being 18 feet, what is the length of the base?

SOLUTION.

Here $P = 9$ lbs., $W = 700$ lbs., and $H = 18$ feet.

Then $B = \frac{W \times H}{P} = \frac{700 \times 18}{9} = 1400$ feet. *Ans.*

EXERCISE.

77. On an inclined plane rising 1 foot in 35 feet, what power acting parallel to the plane will support a weight of 17500 lbs.? *Ans.* 500 lbs.
78. On an inclined plane rising 9 feet in 100 feet, what power acting parallel to the plane will sustain a weight of 4237 lbs.? *Ans.* $381\frac{33}{100}$ lbs.
79. On an inclined plane whose height is 11 feet and base 900 feet, what power acting parallel to the base will sustain a weight of 27900 lbs.? *Ans.* 341 lbs.
80. On an inclined plane rising 7 feet in 91 feet, what weight will be supported by a power of 1300 lbs., acting parallel with the plane? *Ans.* 16900 lbs.
81. On an inclined plane a power of 2 lbs., acting parallel to the plane, sustains a weight of 10 lbs.—what is the inclination of the plane? *Ans.* Plane rises 1 foot in 5 feet.
82. On an inclined plane a power of 7 lbs., acting parallel to the base, sustains a weight of 147 lbs.—if the base of the plane be 17 feet what will its height be? *Ans.* $\frac{17}{21}$ feet.
83. On an inclined plane rising 2 feet in 109 feet, what weight will be sustained by a power of 17 lbs., acting parallel to the plane? *Ans.* $926\frac{1}{2}$ lbs.
84. On an inclined plain a power of $4\frac{3}{7}$ lbs., sustains a weight of $223\frac{4}{11}$ lbs.; the power acting parallel to the plain, what is the degree of inclination? *Ans.* Plane rises 341 feet in 17199 feet.
85. What weight will be supported by a power of 60 lbs., acting parallel to the base of an inclined plane whose height is 7 feet and base 15 feet? *Ans.* $128\frac{4}{7}$ lbs.

THE WEDGE.

118. The wedge is merely a movable inclined plane or a double inclined plane, *i. e.*, two inclined planes joined together by their bases.

119. The wedge is worked either by *pressure* or by *percussion.*

NOTE.—When the wedge is worked by percussion, the relation between the power and weight cannot be ascertained since the force of percussion differs so completely from continued forces as to admit of no comparison with them.

120. *In the wedge the conditions of equilibrium are that the power is to the weight as half the width of the back of the wedge is to its length.*

NOTE 1.—Unlike all other mechanical powers, the practical use of the wedge depends on friction, as, were it not prevented by friction, the wedge would recoil at every stroke.

NOTE 2.—Razors, knives, scissors, chisels, awls, pins, needles, &c., are examples of the application of the wedge to practical purposes.

121. *For the wedge, let* P = *power or pressure*, W = *the weight*, L = *the length of the wedge, and* B = *the width of the back.*

Then $P : W :: \frac{1}{2} B : L$. *Hence* $P = \frac{W \times \frac{1}{2} B}{L}$ and $W = \frac{P \times L}{\frac{1}{2} B}$.

EXAMPLE 86.—The length of a wedge is 24 inches, and its thickness at the back 3 inches, what weight would be raised by a pressure of 750 lbs.?

SOLUTION.

Here $P = 750$ lbs., $L = 24$ inches, and $\frac{1}{2} B = 1\frac{1}{2}$ inch.

Then $W = \frac{P \times L}{\frac{1}{2}B} = \frac{750 \times 24}{1\frac{1}{2}} = 750 \times 16 = 12000$ lbs. *Ans.*

EXAMPLE 87.—In a wedge, the length is 17 inches, thickness of back 2 inches, and the weight to be raised is 11000 lbs. Required the pressure to be applied?

SOLUTION.

Here $W = 11000$, $L = 17$ inches, and $\frac{1}{2} B = 1$ inch.

Then $P = \frac{W \times \frac{1}{2}B}{L} = \frac{11000 \times 1}{17} = 647\frac{1}{17}$ lbs. *Ans.*

EXERCISE.

88. The length of a wedge is 30 inches and the thickness of its back 1 inch, what weight will be raised by a pressure of 97 lbs.? *Ans.* 5820 lbs.
89. The length of a wedge is 19 inches and the thickness of its back 4 inches, what pressure will be required to raise a weight of 864 lbs.? *Ans.* $90\frac{18}{19}$ lbs.
90. The length of a wedge is 23 inches and the thickness of its back 3 inches—with this instrument what pressure would be required to raise a weight of 1771 lbs.? *Ans.* $115\frac{1}{2}$ lbs.

THE SCREW.

122. The screw is a modification of the inclined plane, and may be regarded as being formed of an inclined plane, wound round a cylinder.

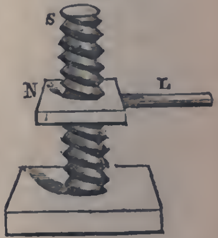
NOTE.—The screw bears the same relation to an ordinary inclined plane that a circular staircase does to a straight one.

123. The threads of the screw are either triangular or square. The distance of a thread and a space when the thread is square, or the distance between two contiguous triangular threads, is called the *pitch*.

124. The screw is commonly worked by pressure against the threads of an external screw, called the *box* or *nut*. The power is applied either to turn the screw while the nut is fixed, or to turn the nut while the screw is kept immovable.

125. In practice, the screw is seldom used as a simple mechanical power, being nearly always combined with some one of the others —usually the lever.

Fig. 13.



126. The conditions of equilibrium between the power and the weight in the screw are the same as for the inclined plane, where the power acts parallel to the base, i. e.,

The power is to the weight as the pitch (i. e. height) is to the circumference of the base (i. e. length of the plane.)

When the screw is worked by means of a lever, the conditions of equilibrium are :—

The power is to the weight as the pitch is to the circumference of the circle described by the power.

127. The efficiency of the screw as a mechanical power may be increased by two methods :

1st. By diminishing the pitch.

2nd. By increasing the length of the lever.

128. *For the screw, let* P = *the power*, W = *the weight*, p = *the pitch, and* l = *length of the lever.*

Then since the lever forms the radius of the circle described by the power, and the circumference of a circle is 3·1416 *times the diameter, and the diameter is twice the radius,* P : W :: p : l × 2 × 3·1416.

$$\text{Hence } P = \frac{W \times p}{l \times 2 \times 3{\cdot}1416},\ W = \frac{P \times l \times 2 \times 3{\cdot}1416}{p} \text{ and } p = \frac{P \times l \times 2 \times 3{\cdot}1416}{W}.$$

NOTE.—The pitch and the length of the lever must be both expressed in units of the same denominations, *i. e.*, both feet, or both inches.

EXAMPLE 91.—What power will sustain a weight of 70000 lbs. by means of a screw having a pitch of $\frac{1}{14}$ of an inch, and the lever to which the power is attached 8 ft. 4 in. in length?

SOLUTION.

Here $W = 70000$ lbs., $p = \frac{1}{14}$ in., and $l = 8$ ft. 4 in $= 100$ in.

Here $P = \frac{W \times p}{l \times 2 \times 3{\cdot}1416} = \frac{70000 \times \frac{1}{14}}{100 \times 2 \times 3{\cdot}1416} = \frac{5000}{628{\cdot}32} = \frac{500000}{62832} = 7{\cdot}957$ lbs. *Ans.*

EXAMPLE 92.—What weight will be sustained by a power of 5 lbs. by means of a screw having a pitch of $\frac{1}{10}$th of an inch, the power lever being 50 inches in length?

SOLUTION.

Here $P = 5$ lbs. $p = \frac{1}{10}$ inch, and $l = 50$ inches.

Then $W = \frac{P \times l \times 2 \times 3{\cdot}1416}{p} = \frac{5 \times 50 \times 2 \times 3{\cdot}1416}{\frac{1}{10}} = \frac{1570{\cdot}8}{\frac{1}{10}} = 15708$ lbs. *Ans.*

EXAMPLE 93.—By means of a screw having a power lever 5 ft. 10 inches in length, a power of 6 lbs. sustains a weight of 80000 lbs.; what is the pitch of the screw?

SOLUTION.

Here $P = 6$ lbs., $W = 80000$ lbs., and $l = 70$ inches.

Then $p = \frac{P \times l \times 2 \times 3{\cdot}1416}{W} = \frac{6 \times 70 \times 2 \times 3{\cdot}1416}{80000} = \frac{2638{\cdot}944}{80000} = {\cdot}0329868$ inches, or about $\frac{33}{1000}$ of an inch. *Ans.*

EXAMPLE 94.—What power will sustain a weight of 96493 lbs. by means of a screw having a pitch of $\frac{3}{17}$th of an inch, the power lever being 25 inches in length?

SOLUTION.

Here $W = 96493$ lbs., $p = \frac{3}{17}$th inch, and $l = 25$.

Then $P = \frac{W \times p}{l \times 2 \times 3{\cdot}1416} = \frac{96493 \times \frac{3}{17}}{25 \times 2 \times 3{\cdot}1416} = \frac{\frac{289479}{17}}{157{\cdot}08} = \frac{17028{\cdot}1764}{157{\cdot}08} = 108{\cdot}403$ lbs. *Ans.*

EXERCISE.

95. What power will support a weight of 87000 lbs. by means of a screw having a pitch of $\frac{5}{29}$th of an inch, the power lever being 6 ft. 3 inches long? *Ans.* 31·83 lbs.

96. What weight will be sustained by a power of 200 lbs. acting on a screw having a pitch of $\frac{3}{50}$th of an inch—the power lever being 15 inches long? *Ans.* 314160 lbs.

97. By means of a screw having a power lever 50 inches in length, a weight of 9000 lbs. is supported by a power of 2 lbs. Required the pitch of the screw.
Ans. ·41888, or rather over $\frac{2}{5}$ of an inch.

98. What power will support a weight of 11900 lbs. by means of a screw having a pitch of $\frac{4}{17}$th of an inch, the power lever being 10 ft. in length? *Ans.* 3·713 lbs.

99. By means of a screw having a power lever 7 ft. 6 inches in length, a power of 10 lbs. supports a weight of 65400; what is the pitch of the screw? *Ans.* ·0864 of an inch.

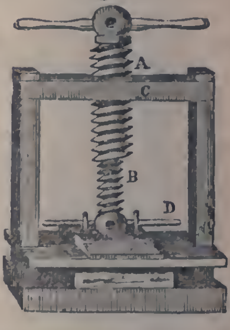
100. What weight will be supported by a power of 50 lbs. acting on a screw with a pitch of $\frac{3}{40}$th of an inch—the power lever being 8 ft. 4 inches in length? *Ans.* 418880 lbs.

THE DIFFERENTIAL SCREW

129. The differential screw, (invented by Dr. John Hunter,) like the differential wheel and axle, acts by diminishing the distance through which the weight is moved in comparison with that traversed by the power.

It consists of two screws of different pitch, working one within the other (Fig. 14), so that at each revolution of the power lever the weight is raised through a space only equal to the *difference* between the pitch of the exterior screw and the pitch of the inner screw. It follows that the mechanical effect of the differential screw is equal to that of a single screw having a pitch equal to the difference of pitch of the two screws.

Fig. 14.



For instance, in Fig. 14, the part B works within the part A. Now, if B have a pitch of $\frac{1}{20}$th of an inch and A a pitch of $\frac{1}{19}$, then at each revolution of the handle the weight will be raised through $\frac{1}{19} - \frac{1}{20} = \frac{1}{380}$ of an inch, and the whole instrument has the same mechanical effect as a single screw having a pitch of $\frac{1}{380}$th of an inch.

130. *For the differential screw, let* P = *power,* W = *weight,* l = *length of lever, and* d = *difference of pitch of the two screws.*

Then $P : W :: d : l \times 2 \times 3{\cdot}1416$.

Hence $P = \dfrac{W \times d}{l \times 2 \times 3{\cdot}1416}$ and $W = \dfrac{P \times l \times 2 \times 3{\cdot}1416}{d}$

EXAMPLE 101.—What power will exert a pressure of 20000 lbs. by means of a differential screw having a power lever 50 inches in length, the exterior screw a pitch of $\frac{3}{11}$ of an inch, and the inner screw a pitch of $\frac{3}{20}$th of an inch?

SOLUTION.

Here $W = 20000$, $l = 50$ in., and $d = \frac{3}{11} - \frac{3}{20} = \frac{60}{220} - \frac{33}{220} = \frac{27}{220}$ of an inch.

Then $P = \frac{W \times d}{l \times 2 \times 3{\cdot}1416} = \frac{20000 \times \frac{27}{220}}{50 \times 2 \times 3{\cdot}1416} = \frac{\frac{27000}{11}}{314{\cdot}16} = \frac{2454{\cdot}545}{314{\cdot}16} = \frac{245454{\cdot}54}{314{\cdot}16}$ $= 7{\cdot}81$ lbs. *Ans.*

EXAMPLE 102.—What pressure will be exerted by a power of 1000 lbs. acting on a differential screw in which the power lever is 75 inches long, the pitch of the exterior screw $\frac{3}{17}$ of an inch, and that of the interior screw $\frac{7}{50}$ of an inch?

SOLUTION.

Here $P = 1000$ lbs., $l = 75$ inches, and $d = \frac{3}{17} - \frac{7}{50} = \frac{150}{850} - \frac{119}{805} = \frac{31}{850}$ of an inch.

Then $W = \frac{P \times l \times 2 \times 3{\cdot}1416}{d} = \frac{1000 \times 75 \times 2 \times 3{\cdot}1416}{\frac{31}{850}} = \frac{471240}{\frac{31}{850}} = \frac{400554000}{31}$ $= 12921096\frac{24}{31}$ lbs. *Ans.*

EXERCISE.

103. What power will exert a pressure of 100000 lbs. by means of a differential screw in which the power lever is 100 inches long, the pitch of the outer screw $\frac{1}{39}$ of an inch, and that of the inner screw $\frac{1}{40}$ of an inch?

Ans. ·102 or about $\frac{1}{10}$ of a lb.

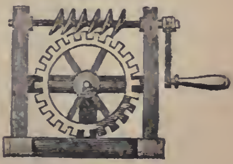
104. What pressure will be exerted by a power of 20 lbs. acting on a differential screw in which the power lever is 50 inches long, the pitch of the exterior screw $\frac{2}{11}$ of an inch, and that of the inner screw $\frac{1}{6}$ of an inch? *Ans.* 345576 lbs.

105. What power will give a pressure of 60000 lbs. by means of a differential screw in which the power lever is 60 inches, the pitch of the outer screw $\frac{3}{20}$, and that of the inner screw $\frac{2}{15}$ of an inch? *Ans.* 2·652 lbs.

THE ENDLESS SCREW.

131. The Endless Screw, Fig. 15, is an instrument formed by combining the screw with the wheel and axle. The teeth of the wheel are set obliquely so as to act as much as possible on the threads of the screw.

Fig. 15.



132. In Fig. 15 each revolution of the handle makes the wheel revolve only through the space of one cog; hence if the whole has 24 cogs, the winch must revolve 24 times in order to make the wheel revolve once.

It follows that in the endless or perpetual screw the conditions of equilibrium are that the power is to the weight as the radius of the axle is to the product of the number of teeth in the wheel multiplied by the length of the winch; i. e., the radius of the circle described by the power.

133. *For the endless screw let* P = *power,* W = *weight,* l = *length of winch or handle,* t = *number of teeth in the wheel, and* r = *radius of axle.*

Then $P : W :: r : l \times t$. Whence $P = \frac{W \times r}{l \times t}$ $W = \frac{P \times l \times t}{r}$.

EXAMPLE 106.—In an endless screw the length of the winch or handle is 25 inches, the wheel has 60 cogs, and the axle to which the weight is attached has a radius of 2 inches. What weight will be sustained by a power of 100 lbs.?

SOLUTION.

Here $P = 100$ lbs., $r = 2$ inches, $l = 25$ inches, and $t = 56$.

Then $W = \frac{P \times l \times t}{r} = \frac{100 \times 25 \times 60}{2} = \frac{150000}{2} = 75000$ lbs. *Ans.*

EXAMPLE 107.—In an endless screw the length of the winch is 20 inches, the wheel has 56 teeth and the radius of the axle is 3 inches. What power will support a weight of 14000 lbs.?

SOLUTION.

Here $W = 14000$ lbs., $r = 3$ inches, $l = 20$ inches, and $t = 60$.

Then $P = \frac{W \times r}{l \times t} = \frac{14000 \times 3}{20 \times 56} = \frac{42000}{1120} = 37\frac{1}{2}$ lbs. *Ans.*

EXERCISE.

108. In an endless screw the length of the winch is 18 inches, the radius of the axle is 2 inches, the wheel has 48 teeth, and the power is 120 lbs. Required the weight. *Ans.* 51840 lbs.

109. What power will support a weight of a million of lbs. by means of an endless screw having a winch 25 inches long, an axle with a radius of 1 inch, and a wheel with 100 teeth? *Ans.* 400 lbs.

110. What weight will be raised by a power of 40 lbs. by means of an endless screw in which the winch is 20 inches long, the radius of the axle 2 inches, and the number of teeth in the wheel 80? *Ans.* 32000 lbs.

134. The theoretical results obtained by the foregoing rules are in practice very greatly modified by several retarding forces. Thus friction has to be taken into account in each of the mechanical powers—the weight of the instrument itself in the lever and in the movable pulley—the rigidity of cordage in the pulley and in the wheel and axle, &c.

FRICTION.

135. Friction aids the power in supporting the weight, but opposes the power in moving the weight, and hence, materially affects the conditions of equilibrium in the mechanical powers.

If P be the power necessary in the absence of all friction, and f the friction, then the weight will be held in equilibrium by any power which is less than $P+f$, or greater than $P-f$.

136. Friction is of two kinds: 1st. Sliding Friction.
2nd. Rolling Friction.

137. The fraction which expresses the ratio between the whole weight and the power necessary to overcome the friction, is called the *coefficient of friction.* The coefficient of sliding friction, in the case of hard bodies, varies from $\frac{1}{7}$ to $\frac{1}{2}$.

138. On a perfectly level road, power is expended only for the purpose of overcoming friction, and on the same road the ratio between the power and the load is constant,—varying on common roads, according to their goodness, from $\frac{1}{18}$ to $\frac{1}{50}$ of the load. On an even railway, however, it is not more than $\frac{1}{150}$ to $\frac{1}{280}$ of the load, according to the dampness or dryness of the rail. On a good macadamized road the coefficient of friction is about $\frac{1}{30}$, so that a horse drawing a load of one ton or 2000 lbs. must draw with a force of $\frac{1}{30}$ of 2000 lbs. or $66\frac{2}{3}$ lbs.; this is called the *force of traction.*

139. Various expedients are in common use for diminishing the amount of friction, such as crossing the grain, when wooden surfaces rub on one another, using surfaces of different materials, as wood on metal, or one kind of

metal on another kind, and annointing the surface with oil, tar, or plumbago. Tallow diminishes the friction by one-half.

The following are the conclusions of Coulomb on the important subject of sliding friction :—

I. Friction is directly proportional to the pressure.

II. Friction between the same two bodies is constant, being uninfluenced by either the extent of surface in contact or the velocity of the motion.

III. Friction is greatest between surfaces of the same material.

IV. Friction varies with the nature of the surface in contact.

The friction between surfaces of wood, newly planed	$= \frac{1}{2}$
The friction between similar metallic surfaces	$= \frac{1}{4}$
The friction of a wooden surface on a metallic surface	$= \frac{1}{5}$
The friction of iron sliding on iron	$= \frac{2}{7}$
The friction of iron sliding on brass	$= \frac{1}{6}$

V. Friction decreases as the surfaces in contact wear. In wood the friction is thus reduced from $\frac{1}{2}$ to $\frac{1}{3}$.

VI. Friction is diminished between wooden surfaces by crossing the fibres. If when the fibres are in the same direction the coefficient of friction is $\frac{1}{2}$, it is diminished to $\frac{1}{4}$ by crossing them.

VII. Friction is greater between rough than between polished surfaces. Hence arises the use of lubricants in machinery. When the pressure is small, the most limpid oils are used. At greater pressures, the more viscid oils are preferred, then tallow, then a mixture of tallow and tar, or tallow and plumbago, then plumbago alone, and in the heaviest machinery *soapstone* has been found to be the most efficacious substance.

Note.—At *very great* velocities the friction is perceptibly lessened; when the pressure is *very greatly* increased, the friction is not increased in proportion.

ROLLING FRICTION.

VIII. Friction caused by one body rolling on another is directly proportional to the pressure, and inversly to the diameter of the rolling body.

That is, if a cylinder rolling along a plane have its pressure doubled, its friction will also be doubled; but if its diameter be doubled, the friction will be only half of what it was.

The friction of a wooden cylinder of 32 inches in diameter rolling upon rollers of wood is $\frac{1}{125}$ of the pressure.

The friction of an iron axle turning in a box of brass and well coated with oil is $\frac{1}{40}$ of the pressure.

CHAPTER IV.

UNIT OF WORK, WORK OF DIFFERENT AGENTS, HORSE POWER OF LOCOMOTIVES, STEAM ENGINES, AND WORK OF STEAM.

UNIT OF WORK.

140. In comparing the work performed by different agents, or by the same agent under different circumstances, it becomes necessary to make use of some definite and distinct unit of work. The unit commonly adopted for this purpose in England and America is the *labor requisite to raise the weight of one pound through the space of one foot.*

Thus in raising 1 lb. through 1 foot, 1 unit of work is performed.

If 2 lbs. be raised 1 ft., or if 1 lb. be raised 2 ft., 2 units of work are performed.

If 7 lbs. be raised through 9 ft., or if 9 lbs. be raised through 7 ft., 63 units of work are performed, &c.

141. *The units of work expended in raising a body of a given weight are found by multiplying the weight of the body in lbs. by the vertical space in feet through which it is raised.*

EXAMPLE 111.—How many units of work are expended in raising a weight of 642 lbs. to a height of 70 ft.?

SOLUTION.

Ans. Units of work $= 642 \times 70 = 44940$.

EXAMPLE 112.—How many units of work are expended in raising a weight of 423 lbs. to a height of 267 ft.?

SOLUTION.

Ans. Units of work $= 423 \times 267 = 112941$.

EXAMPLE 113.—How many units of work are expended in raising 11 tons of coal from a pit whose depth is 140 ft.?

SOLUTION.

Here, 11 tons $= 11 \times 2000 = 22000$ lbs.
Then $22000 \times 140 = 3080000$. *Ans.*

EXAMPLE 114.—How many units of work are expended in raising 7983 gallons of water to the height of 79 ft.?

SOLUTION.

Here, since a gallon of water weighs 10 lbs., 7983 gals. $= 79830$ lbs.
Then units of work $= 79830 \times 79 = 6306570$. *Ans.*

EXAMPLE 115.—How many units of work are expended in raising 60 cubic feet of water from a well whose depth is 90 feet?

SOLUTION.

Since a cubic foot of water weighs $62\frac{1}{2}$ lbs., 60 cubic feet weigh $62\frac{1}{2} \times 60 =$ 3750 lbs.
Then units of work $= 3750 \times 99 = 337500$. *Ans.*

EXERCISE.

116. How much work would be required to pump 60000 gallons of water from a mine whose depth is 860 feet?
Ans. 516000000 units.

117. How many units of work would be expended in pumping 8000 cubic feet of water from a mine whose depth is 679 feet?
Ans. 339500000 units.

118. How much work would be expended in raising the ram of a pile driving engine—the ram weighing 2 tons, and the height to which it is raised being 29 feet?
Ans. 116000 units.

119. How much work would be required to raise 17 tons of coals from a mine whose depth is 300 feet?
Ans. 10200000 units.

120. How much work would be expended in raising 600 cubic feet of water to the height of 293 feet?
Ans. 10987500 units.

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142. The most important sources of laboring force are *animals, water, wind,* and *steam.* The laboring force of animals is modified by various circumstances, the most important of which are the duration of the labor, and the mode by which it is applied. The following table shows the amount of effective work that can be performed under different circumstances by the more common living agents:

TABLE.

SHEWING THE WORK DONE PER MINUTE BY VARIOUS AGENTS.

Duration of labor eight hours per day.

Horse	33000	units
Mule	22000	"
Ass	8250	"
Man, with wheel and axle	2600	"
" drawing horizontally	3200	"
" raising materials with a pulley	1600	"
" throwing earth to the height of 5 feet.	560	"

Man, working with his arms and legs as in rowing	4000 units
" raising water from a well with a pail and rope............	1054 "
" raising water from a well with an upright chain pump................	1730 "

NOTE.—The work assigned by Watt, to the horse per minute was 33000 units, but this is known to be about $\frac{1}{3}$ too great. A horse of average strength performs about 22000 units of work per minute. The number given in the table, however, is still used in all calculations in civil engineering.

EXAMPLE 121.—How many cubic feet of earth, each weighing 100 lbs., will a man throw to the height of 5 feet in a day of 8 hours?

SOLUTION.

Since (by the table) a man throwing earth to the height of 5 ft., does 560 units of work per minute—and from the example he works $8 \times 60 = 480$ minutes.

Units of work done in the day $= 560 \times 480$.

Units of work required to throw 1 cubic foot to height of 5 feet $= 100 \times 5$.

Then $\frac{560 \times 480}{100 \times 5} = 537\frac{3}{7}$ cubic feet. *Ans.*

EXAMPLE 122.—How many gallons of water will a man raise in a day of 8 hours from a well whose depth is 70 feet—using a pail and rope?

SOLUTION.

Units of work $= 1054 \times 60 \times 8$; work required to raise 1 gal. $= 10 \times 70$.

Then number of gallons $= \frac{1054 \times 60 \times 8}{10 \times 70} = 722\frac{26}{35}$. *Ans.*

EXAMPLE 123.—How many gallons of water can a man raise by means of a chain pump in a day of 8 hours from the depth of 80 feet?

SOLUTION.

Units of work performed by the man $= 1730 \times 60 \times 8$.

Units of work required to raise 1 gal. of water $= 10 \times 80$.

The number of gallons $= \frac{1730 \times 60 \times 8}{10 \times 80} = 1038$. *Ans.*

EXAMPLE 124.—How many tons of earth will a man working with a wheel and axle raise in a day of 8 hours from a depth of 87 feet?

SOLUTION.

Units of work performed by the man $= 2600 \times 60 \times 8$.

Units of work required to raise 1 ton to height of 87 ft. $= 2000 \times 87$.

Tons raised $= \frac{2600 \times 60 \times 8}{2000 \times 87} = 7\frac{5}{29}$. *Ans.*

EXAMPLE 125.—How many gallons of water per hour will an engine of 7 horse power raise from a mine whose depth is 110 feet?

SOLUTION.

Units of work in one horse power = 33000 per minute.
Units of work in 7 horse power = 33000 × 7.
Units of work performed by the engine per hour = 33000 × 7 × 60.
Units of work required to raise 1 gallon of water to the height of 110 ft. = 10 × 110.

Hence number of gallons $= \frac{33000 \times 7 \times 60}{10 \times 110} = 1260$. *Ans.*

EXAMPLE 126.—How many horse power will it require to raise 22 tons of coals per hour from a mine whose depth is 360 feet?

SOLUTION.

Weight of coals to be raised = 22 tons = 44000 lbs.
Units of work required per hour = 44000 × 360.
Units of work in one horse power per hour = 33000 × 60.

Hence H. P. $= \frac{44000 \times 360}{33000 \times 60} = 8$. *Ans.*

EXAMPLE 127.—How many cubic feet of water will an engine of 15 horse power pump each hour from a mine whose depth is 900 feet?

SOLUTION.

Units of work performed by engine per hour = 33000 × 60 × 15.
Units of work required to raise 1 cubic foot = 62·5 × 900.

Hence, number of cubic feet $= \frac{33000 \times 60 \times 15}{62 \cdot 5 \times 900} = 528$. *Ans.*

EXAMPLE 128.—What must be the horse power of an engine in order that working 12 hours per day it may supply 2300 families with 50 gallons of water each per day—taking the mean height to which the water is raised as 80 feet, and assuming that $\frac{1}{6}$ of the work of the engine is lost in transmission?

SOLUTION.

Weight of water pumped per day = 2300 × 50 × 10.
Units of work required daily = 2300 × 50 × 10 × 80.
Units of work in one horse power per day = 3300 × 12 × 60.
But since $\frac{1}{6}$ of the work of the engine is lost in transmission.
Useful work of one H. P. per day = $\frac{5}{6}$ × 33000 × 12 × 60.

Hence, H. P. $= \frac{2300 \times 50 \times 10 \times 80}{\frac{5}{6} \times 33000 \times 12 \times 60} = 4 \cdot 64$. *Ans.*

EXERCISE.

129. How many cubic feet of earth, each weighing 100 lbs., will a man raise by means of a pulley from a depth of 30 feet in a day of 8 hours? *Ans.* 256 cubic feet.

130. How many cubic feet of water per hour will an engine of 20 H. P. raise from a mine whose depth is 450 feet, assuming that $\frac{1}{8}$ of the work of the engine is lost in transmission? *Ans.* 1126$\frac{2}{3}$ cubic feet.

131. What must be the H. P. of an engine in order that it may raise 11 tons of material per hour from a depth of 700 ft? *Ans.* 7·77 H. P.

132. A forge hammer weighing 890 lbs. makes 50 lifts of 4 feet each per minute—what must be the horse power of the engine that works the hammer? *Ans.* H. P. = 5·39

133. An engine of 8 horse power works a forge hammer, causing it to make 50 lifts per minute, each to the eight of 6 feet. What is the weight of the hammer? *Ans.* 880 lbs.

134. An engine of 8 horse power gives motion to a forge hammer, which weighs 300 lbs., and makes 30 lifts per minute of 2 feet each; and at the same time raises 2 tons of coal per hour from the bottom of a mine. Required the depth of the mine. *Ans.* 3690 feet.

NOTE.—The work of the engine = 33000 × 8 units per minute. From this subtract the units of work required by the hammer; the remainder will be the work expended per minute in raising the coal. Multiplying this by 60 gives us the work required per hour for the coal; and this last is the product of the weight in lbs. by the depth in feet, of which the former is given.

WORK EXPENDED IN MOVING A CARRIAGE OR RAILWAY TRAIN ALONG A HORIZONTAL PLANE.

143. In moving a carriage, &c., along a level plane, a certain amount of power is expended in overcoming the friction of the road. This is rolling friction, and amounts as before stated (Art. 138,) to form $\frac{1}{50}$ to $\frac{1}{18}$ of the entire load on common roads, and form $\frac{1}{280}$ to $\frac{1}{150}$ of the load on railway tracks. In the case of railway trains, friction is usually taken as 7 lbs. per ton of 2000 lbs.

144. In running carriages of any description, work is employed to overcome the resistances. These resistances are:—

1st. *Friction*—which on the same road and with the same load is the same for all velocity.

2nd. *Ascent of inclined planes*—in which, since the load has to be lifted vertically through the height of the plane the work is the same, whatever may be the velocity of the motion.

3rd. *The Resistance of the Atmosphere*—which depends upon the extent of surface, and increases as the square of the velocity.

145. When a railway train is set in motion, the work of the locomotive engine at first far exceeds the work of resistances, and the motion is consequently rapidly accelerated. But as the velocity of the train increases, the atmospheric resistance also increases, and with such rapidity as very soon to equalize the work of resistances to the work of the locomotive. When this occurs, i. e., when the work applied by the locomotive is exactly equal to the continued work of resistances (atmospheric resistance and friction), the velocity of the train will be uniform. In this case the train is said to have attained its greatest or maximum speed.

146. The *traction* or force with which an animal pulls depends upon the rate of his motion. A horse, for example, moving only 2 miles an hour, can draw with a far greater force than when running at the rate of 6 miles an hour. The following table shows the relation between the speed and the traction of a horse:

TABLE OF TRACTION OF A HORSE.

	Speed.			*Traction.*
A horse moving	2	miles per hour,	can draw with a force of	166 lbs.
"	3	"	"	125 "
"	3½	"	"	104 "
"	4	"	"	83 "
"	4½	"	"	62½ "
"	5	"	"	41¾ "

EXAMPLE 135.—What gross load will a horse draw travelling at the rate of four miles per hour on a road whose friction is $\frac{1}{20}$ of the whole load?

SOLUTION.

Here from the table the traction is 83 lbs., which by the conditions of the question is $\frac{1}{20}$ of the gross load.
Hence load $= 83 \times 20 = 1660$ lbs. *Ans.*

EXAMPLE 136.—At what rate will a horse draw a gross load of 1800 lbs. on a road whose coefficient of friction is $\frac{1}{18}$?

SOLUTION.

Here traction $= \frac{1800}{18} = 100$ lbs., whence by the table the rate must be rather over 2½ miles per hour.

EXAMPLE 137.—If a horse draw a load of 2500 lbs. upon a road whose coefficient of friction is $\frac{1}{30}$, what traction will he exert, and how many units of work will he perform per minute?

SOLUTION.

Here, traction $= \frac{2500}{30} = 83\frac{1}{3}$ lbs., and hence he moves at a rate of four miles per hour.

The distance moved per minute $= \frac{4 \times 5280^*}{60} = 352$ feet.

Hence units of work $= 83\frac{1}{3} \times 352 = 29333\frac{1}{3}$. *Ans.*

EXAMPLE 138.—What must be the effective horse power of a locomotive engine to carry a train weighing 70 tons upon a level rail at the steady rate of 40 miles per hour, neglecting atmospheric resistance and taking $\frac{1}{200}$ as the coefficient of friction?

SOLUTION.

Here, weight of train $= 70$ tons $= 140000$ lbs.

Space passed over per minute $= \frac{40}{60}$ miles $= \frac{40 \times 5280}{60} = 3520$ feet.

Work of friction to one foot $= \frac{1}{200}$ of $140000 = \frac{140000}{200} = 700$ units.

Work of friction per minute $= 700 \times 3520 = 2464000$ units.
Units of work in one H. P. $= 33000$.

Therefore H. P of locomotive $= \frac{700 \times 3520}{33000} = \frac{2464000}{33000} = 74\cdot66$. *Ans.*

EXAMPLE 139.—A train weighing 120 tons is carried with a uniform velocity of 30 miles per hour along a level rail; assuming the friction to be 11 lbs. per ton, and neglecting the resistance of the atmosphere, what is the horse power of the locomotive?

SOLUTION.

Space passed over per minute $= \frac{30}{60}$ miles $= \frac{30 \times 5280}{60} = 2640$ feet.

Work of friction to each foot $= 120 \times 11 = 1320$ units.
Work of friction per minute $= 1320 \times 2640 = 3484800$ units.

Hence H. P. $= \frac{3484800}{33000} = 105\cdot6$. *Ans.*

EXAMPLE 140.—At what rate per hour will a train weighing 90 tons be drawn by an engine of 80 horse power, neglecting the resistance of the atmosphere and taking $\frac{1}{250}$ as the coefficient of friction?

SOLUTION.

Work done by the engine per hour $= 33000 \times 60 \times 80$.
Weight of train in lbs. $= 90 \times 2000 = 180000$.

Units of work required to move the train through 1 foot $= \frac{1}{250}$ of $180000 = 720$.
Work expended in moving the train through 1 mile $= 720 \times 5280$.

$\therefore$ Number of miles per hour $= \frac{33000 \times 60 \times 80}{720 \times 5280} = 41\cdot66$. *Ans.*

EXAMPLE 141.—A train moves on a level rail with the uniform speed of 35 miles per hour; assuming the H. P. of the locomotive to be 50, the friction equal to 9 lbs. per ton, and neglecting atmospheric resistance, what is the gross weight of the train?

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* 5280 is the number of feet in one mile.

SOLUTION.

Work of engine per hour = $33000 \times 60 \times 50$.
Feet moved over per hour = 35×5280.
Work expended per hour in moving 1 ton = $35 \times 5280 \times 9$.

$\therefore$ Weight of train in tons $= \dfrac{33000 \times 60 \times 50}{35 \times 5280 \times 9} = 59{\cdot}523$. *Ans.*

EXAMPLE 142.—In what time will an engine of 100 H. P. move a train of 90 tons weight through a journey of 80 miles along a level rail, assuming friction to be equal to 10 lbs. per ton, and neglecting atmospheric resistance?

SOLUTION.

Work expended in moving the train through 1 foot = $90 \times 10 = 900$ units.
Work expended on whole journey in moving the train = $900 \times 5280 \times 80$.
Work of engine per minute = 33000×100.

$\therefore$ Number of minutes $= \dfrac{900 \times 5280 \times 80}{33000 \times 100} = 115\frac{1}{5}$ minutes = 1 hour $55\frac{1}{5}$ minutes.

EXERCISE.

143. What gross load will a horse draw travelling at the rate of 2 miles per hour on a road whose coefficient of friction is $\frac{1}{18}$? *Ans.* 2988 lbs.

144. What must be the H. P. of a locomotive in order that it may draw a train whose gross weight is 130 tons, at the uniform speed of 25 miles per hour, allowing the friction to be 7 lbs per ton, and neglecting atmospheric resistance? *Ans.* H. P. 60·66.

145. A train weighs 75 tons, and moves with the uniform speed of 30 miles per hour on a level rail; taking $\frac{1}{280}$ as the coefficient of friction, and neglecting the resistance of the atmosphere, what is the horse power of the engine? *Ans.* H. P. = 48.

146. In what time will an engine of 160 H. P., moving a train whose gross weight is 110 tons, complete a journey of 150 miles taking friction to be equal to 7 lbs. per ton, neglecting atmospheric resistance, and assuming the rail to be on a level plane throughout? *Ans.* 1 hour $55\frac{1}{4}$ minutes.

147. At what rate per hour will a horse draw a load whose gross weight is 2200 lbs. on a road whose coefficient of friction is $\frac{1}{20}$? *Ans.* Rather over $3\frac{1}{4}$ miles per hour.

148. From the table given (Art. 145) ascertain at what rate per hour a horse must travel, when drawing a load, in order to do the greatest amount of work? *Ans.* 3 miles per hour.

149. At what rate per hour will a locomotive of 50 H. P. draw a train whose gross weight is 70 tons, neglecting atmospheric resistance, taking $\frac{1}{200}$ as the coefficient of friction, and assuming the rail to be level? *Ans.* 26.78 miles.

147 When a body moves through the atmosphere or any other fluid, it encounters a resistance which increases:

1st. In proportion to the surface of the moving body;
2nd. In proportion to the square of the velocity.

Thus 1st. If a board presenting a surface of 1 sq. foot in moving through the air meet with a certain resistance, a board having a surface of 2 sq. feet will meet with double that resistance; a board having a surface of 3 square feet will meet with three times that resistance, &c.

2nd. If a body moving 2 miles per hour, meet with a certain resistance, a body of the same size moving 4 miles per hour will meet with $(\frac{4}{2})^2$, or 2^2, or 4 times that resistance.

If the velocity be increased 3 times; i. e., to 6 miles per hour, the resistance will be increased 9 times (i. e., 3^2 times).
If the velocity be increased 7 times (i. e., to 14 miles per hour,) the resistance will be increased 7^2 times, i. e., 49 times, &c.

148. In the case of railway trains, the atmospheric resistance is about 33 lbs, when the train is moving at the rate of 10 miles per hour. It has been found, however, by recent experiment, that the atmospheric resistance encountered by a train in motion depends very much upon the length of the train.

EXAMPLE 150.—When a train is moving at the rate of 10 miles per hour, it encounters an atmospheric resistance of 33 lbs.; what will be the resistance of the atmosphere when the train moves at the rate of 50 miles per hour?

SOLUTION.

Here the velocity increases $\frac{50}{10}$ times, i. e., 5 times.

Hence the resistance increases 5^2 times = 25 times.

∵ Resistance $= 33 \times 25 = 825$ lbs., i. e., 825 units of work are expended every foot in overcoming the atmospheric resistance.

EXAMPLE 151.—If a train moving 7 miles per hour meet with an atmospheric resistance equal to 5 lbs., what resistance will it encounter if its speed be increased to 49 miles per hour?

SOLUTION.

Here the velocity increases 7 times, (*i. e.*, $\frac{49}{7}$).

Hence the resistance increases 7^2 times = 49 times.
∵ Resistance $= 5 \times 49 = 245$ lbs., i. e., 245 units of work are expended every foot in overcoming the atmospheric resistance.

EXAMPLE 152.—If a railway train moving at the rate of 10 miles per hour encounters an atmospheric resistance of 33 lbs.; what must be the horse power of the locomotive in order that the train may move 60 miles per hour, neglecting friction and assuming the rail to be level?

SOLUTION.

Here the velocity is increased 6 times, since $\frac{60}{10} = 6$.

Then the resistance is increased 36 times [Art. 147.]
Hence atmospheric resistance $= 33 \times 36 = 1188$ lbs.; i. e., 1188 units of work are expended in moving the train through 1 ft.

Number of feet train moves through in a minute $= \frac{60 \times 5280}{60} = 5280$.

Units of work required per minute to overcome atmospheric resistance $= 1188 \times 5280$.

$\therefore$ H. P. of locomotive $= \frac{1188 \times 5280}{33000} = 190{\cdot}08$. *Ans.*

EXAMPLE 153.—What must be the H. P. of a locomotive to move a train at the rate of 40 miles per hour on a level rail, taking atmospheric pressure as usual, (i. e., 33 lbs. when a train moves 10 miles per hour,) and neglecting friction?

SOLUTION.

Here velocity increases 4 times, and hence resistance increases 16 times.
Then resistance encountered $= 33 \times 16 = 528 =$ units of work required per foot.
Feet moved over per hour $= 5280 \times 40$; hence units of work per hour $= 5280 \times 40 \times 528$.

Therefore H. P. $= \frac{528 \times 40 \times 5280}{33000 \times 60} = 56{\cdot}32$. *Ans.*

EXAMPLE 154.—What must be the H. P. of a locomotive to draw a train whose gross weight is 80 tons, along a level rail, with the uniform velocity of 40 miles per hour, taking atmospheric resistance and friction as usual?

SOLUTION.

Feet passed over per minute $= \frac{40 \times 5280}{60} = 3520$.

Work of friction per minute $= 80 \times 7 \times 3520 = 1971200$ units.
Work of atmospheric resistance $= 33 \times 16 \times 3520 = 1858509$ units.

Therefore H. P. $= \frac{\text{Work of friction} \times \text{work of atmospheric resistance}}{\text{Work of one H. P.}}$

$= \frac{1971200 + 1858560}{33000} = \frac{3829760}{33000} = 116{\cdot}053$. *Ans.*

EXAMPLE 155.—What must be the H. P. of a locomotive to draw a train, whose gross weight is 125 tons, along a level rail, with the uniform velocity of 42 miles per hour, taking friction as usual, and assuming that the atmospheric resistance encountered by the train is equal to 10 lbs., when moving at the rate of 7 miles per hour?

SOLUTION.

Feet moved over per minute $= \frac{42 \times 5280}{60} = 3696$.

Work of friction per minute $= 125 \times 7 \times 3696 = 3234000$ units.
Work of atmospheric resist. per min. $= 10 \times 36 \times 3696 = 1330560$ units

Then H. P. $= \frac{\text{Work of friction} \times \text{work of atmospheric resistance}}{\text{Work of one H. P.}} =$

$\frac{3234000 + 1330560}{33000} = \frac{4564560}{33000} = 138{\cdot}32$. *Ans.*

EXERCISE.

156. If a train encounters an atmospheric resistance of 8 lbs., when moving at the rate of 5 miles per hour, what resistance will it encounter when its speed is increased to 45 miles per hour? *Ans.* 648 lbs.

157. What must be the H. P. of a locomotive to draw a train at the rate of 30 miles per hour on a level rail, assuming that the atmospheric resistance is equal to 9 lbs., when the train moves 6 miles per hour, and neglecting friction? *Ans.* H. P. = 18.

158. What must be the H. P. of a locomotive to draw a train weighing 140 tons along a level rail with the uniform velocity of 36 miles per hour, taking friction as 7 lbs. per ton, and the resistance of the atmosphere 12 lbs., when the train moves 9 miles per hour? *Ans.* H. P = 112·512.

159. A train weighing 200 tons moves along a level rail with a uniform speed of 30 miles per hour; what is the H. P. of the engine—friction and atmospheric resistance being as usual? *Ans.* H. P. = 135·76.

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149. If a body be moved along a surface without friction or atmospheric resistance, the units of work performed are found by multiplying the weight of the body in lbs. by the vertical distance in feet through which it is raised.

Thus, if a body weighing 12 lbs. be moved 200 ft. along an inclined plane having a rise of 19 feet in 100, the units of work performed will be $12 \times 19 \times 2 = 456$, because in moving up the plane 200 feet, the body is raised through $19 \times 2 = 38$ feet.

150. When a train is moving along an inclined plane, and the inclination is not very great, the pressure on the plane is very nearly equal to the weight of the body. Hence we find the work due to friction by Arts. 143–146, the work due to atmospheric resistance by Art. 148, and the work due to gravity by Art. 149.

EXAMPLE 160.—A train weighing 90 tons is drawn up a gradient having a rise of 3 feet in every 1000 feet, with the uniform speed of 40 miles per hour—neglecting friction and atmospheric resistance, what is the H. P. of the engine?

SOLUTION.

Weight of train in lbs. $= 90 \times 2000 = 180000$.

Feet travelled per minute $= \dfrac{40 \times 5280}{60} = 3520$.

Vertical distance moved through per minute $= \frac{3}{1000}$ of $3520 = 10{\cdot}56$ ft.
Units of work due to gravity per minute $= 10{\cdot}56 \times 180000$.

$\therefore$ H. P. $= \dfrac{10{\cdot}56 \times 180000}{33000} = 57{\cdot}6$. *Ans.*

EXAMPLE 161.—A train weighing 140 tons moves up a gradient having a rise of 3 feet in 1100 feet, with the uniform velocity of 36 miles per hour—neglecting atmospheric resistance and taking friction as usual, what is the H. P. of the locomotive?

SOLUTION.

Here weight of train in lbs. $= 140 \times 2000 = 280000$; and speed per minute $= \frac{36 \times 5280}{60} = 3168$ feet.

The units of work due per minute to friction $= 140 \times 7 \times 3168 = 3104640$.
Height to which train is raised per minute $= \frac{3}{1100}$ of $3168 = 8{\cdot}64$ feet.
Then units of work due per minute to gravity $= 8{\cdot}64 \times 280000 = 2419200$.

$\therefore$ H. P. $= \frac{\text{work due gravity} + \text{work due friction}}{\text{Work of one H. P.}} = \frac{3104640 + 2419200}{33000} = \frac{5623840}{33000} = 167{\cdot}389$. *Ans.*

EXAMPLE 162.—A train weighing 100 tons moves up a gradient with a uniform velocity of 30 miles per hour, the rise of the plane being 3 feet in 1000 feet, and taking friction and atmospheric resistance as usual, what is the H. P. of the locomotive?

SOLUTION.

Here weight of train in lbs. $= 100 \times 2000 = 200000$; space passed per minute $= \frac{30 \times 5280}{60} = 2640$ feet, and elevation of train per minute $= \frac{3}{1000}$ of $2640 = 7{\cdot}92$ feet.

Work of friction per minute $= 100 \times 7 \times 2640 = 1848000$ units.
Work of atmospheric resistance per minute $= 33 \times 9 \times 2640 = 784080$ units.
Work of gravity per minute $= 7{\cdot}92 \times 200000 = 1584000$ units.

Then H. P. $= \frac{\text{Work due to fric. per min.} + \text{work due to atmos. resist. per min.} + \text{work due to grav. per min.}}{\text{Units of work in one H. P.}}$

$\therefore$ H. P. $= \frac{1848000 + 784080 + 1584000}{33000} = \frac{421{,}6080}{33000} = 127{\cdot}76$. *Ans.*

EXAMPLE 163.—A train weighing 130 tons descends a gradient having a rise of 7 feet in 2000 feet, with the uniform velocity of 60 miles per hour—taking atmospheric resistance as usual, and the coefficient of friction $\frac{1}{200}$, what is the horse power of the locomotive?

SOLUTION.

Here weight of train in lbs. $= 130 \times 2000 = 260000$; space passed over per minute $= \frac{60 \times 5280}{60} = 5280$ feet.; increase in the velocity $= \frac{60}{10} = 6$; and vertical fall of train per minute $= \frac{7}{2000}$ of 5280 feet. $= 18{\cdot}48$ ft.

Then work of friction per minute $= \frac{1}{200} \times 260000 \times 5280 = 1300 \times 5280 = 6864000$ units.

Work of atmospheric resistance per minute $= 33 \times 36 \times 5280 = 6272640$ units.
Work of gravity per minute $= 18{\cdot}48 \times 260000 = 4804800$ units.
Then, since the train descends the gradient, *gravity acts with* the engine.

Hence H. P. $= \frac{\text{Work of friction} + \text{work of atmos. resist.} - \text{work of gravity}}{\text{Work of one H. P.}}$

$\therefore$ H. P. $= \frac{6864000 + 6272640 - 4804800}{33000} = \frac{8331840}{33000} = 252{\cdot}48$. *Ans.*

EXAMPLE 164.—A train weighing 80 tons moves along a gradient with the uniform speed of 40 miles per hour—assuming the inclination of the gradient to be 3 ft. in 1000 ft., and taking friction and atmospheric resistance as usual, what will be the H. P. of the locomotive :

1st. If the train move up the gradient, and
2nd. If the train move down the gradient?

SOLUTION.

Here weight of train in lbs. $= 80 \times 2000 = 160000$; space passed over per minute $= \frac{40 \times 5280}{60} = 3520$ ft.; velocity is increased $\frac{40}{10} = 4$ times, and vertical ascent or descent of train $\frac{3}{1000}$ of $3520 = 10{\cdot}56$ ft.

Work of friction $= 80 \times 7 \times 3520 = 1971200$ units per minute.
Work of atmospheric resistance $= 33 \times 16 \times 3520 = 1858560$ units per min.
Work of gravity $= 10{\cdot}56 \times 160000 = 1689600$ units per minute.

$$\text{Then H. P.} = \frac{\text{Work of friction} + \text{work of atmos. resist.} \pm \text{work of gravity.}}{\text{Work of one H. P.}}$$

$$\text{Train ascending, H. P.} = \frac{1971200 + 1858560 + 1689600}{33000} = \frac{5519360}{33000} = 167{\cdot}253.$$

$$\text{Train descending, H. P.} = \frac{1971200 + 1858560 - 1689600}{33000} = \frac{2140160}{33000} = 64{\cdot}853.$$

EXAMPLE 165.—A train weighing 110 tons ascends a gradient having a rise of $\frac{1}{8}$ in 100—taking friction as usual, and neglecting atmospheric resistance, what is the maximum speed the train will attain if the H. P. of the locomotive be 120?

SOLUTION.

Here weight of train in lbs. $= 110 \times 2000 = 220000$.
Work of friction in one mile $= 110 \times 7 \times 5280 = 4065600$ units.
Work of gravity in one mile $= \frac{1}{800}$ of $5280 = 6{\cdot}6 \times 220000 = 1452000$ units.
Total work of resistance in one mile $= 4065600 + 1452000 = 5517600$ units.
Total work of engine per hour $= 33000 \times 60 \times 120 = 237600000$ units.

$$\therefore \text{Number of miles per hour} = \frac{237600000}{5517600} = 43{\cdot}06 \ \textit{Ans.}$$

EXAMPLE 166.—If a horse exert a traction of 120 lbs., what gross load will he pull up a hill whose rise is 17 feet. in 1000 ft., assuming the coefficient of friction to be $\frac{1}{10}$?

SOLUTION.

Work of horse in moving the load over 1000 ft. $= 120 \times 1000 = 120000$ units.
Work of friction in moving 1 lb. over 1000 ft. $= 1 \times \frac{1}{10} \times 1000 = 100$ units.
Work of gravity in moving 1 lb. over 1000 ft $= 1 \times 17 = 17$ units.
Total work in moving 1 lb. over 1000 ft = work of friction + work of gravity $= 100 + 17 = 117$ units.
$\therefore$ Number of lbs. drawn by horse $= \frac{120000}{117} = 1025{\cdot}641$. *Ans.*

EXAMPLE 167.—What backward pressure is exerted by a horse in going down a hill which has a rise of 7 feet in a 100, with a load whose gross weight is 2000 lbs., assuming $\frac{1}{35}$ to be the coefficient of friction?

SOLUTION,

Here on a level plane the friction would be $\frac{1}{35}$ of 2000 lbs. = 57·14 lbs. = units of work for each foot.

Work of gravity = $\frac{7}{100}$ of 2000 = 140 units to each foot.
Therefore, the backward pressure is 140 — 57·14 = 82·86 lbs. *Ans.*

EXERCISE.

168. What backward pressure will a horse exert in going down a hill which has a rise of 9 feet in 100, with a load whose gross weight is 1200 lbs., assuming the coefficient of friction of the road to be $\frac{1}{30}$? *Ans.* 68 lbs.

169. What gross load will a horse exerting a traction of 150 lbs. draw up a hill whose inclination is 3 in 100—assuming the coefficient of friction to be $\frac{1}{16}$? *Ans.* 1551·72 lbs.

170. What will be the maximum speed attained by a train weighing 200 tons, drawn by a locomotive of 160 H. P. up a gradient having a rise of $\frac{1}{6}$ in 100—taking friction as usual and neglecting atmospheric resistance? *Ans.* 29·032 miles per hour.

171. A train weighing 88 tons moves up a gradient having a rise of $\frac{1}{3}$ in 100 with the uniform velocity of 20 miles per hour —taking friction and atmospheric resistance as usual, what is the H. P. of the locomotive? *Ans.* H. P. = 71·182.

172. A train weighing 95 tons descends a gradient having a fall of $\frac{3}{4}$ in 1000 with the uniform speed of 40 miles per hour —taking friction and atmospheric resistance as usual, what is the H. P. of the locomotive? *Ans* H. P. = 113·742.

173. A train weighing 125 tons moves along a gradient having a rise of $\frac{1}{4}$ in the 100 with the uniform speed of 25 miles per hour—taking friction and atmospheric resistance as usual, what is the H. P. of the engine,

1st. When the train ascends the gradient?

2nd. When the train descends the gradient?

Ans. Going up, H. P. = 113·75; going down, H. P. = 30·416.

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151. For finding the H. P., maximum speed, weight of train, &c., as in the foregoing examples, by representing the variable quantities, such as weight, rate of motion, inclination of plane, &c., by letters, we may easily deduce formulas by means of which the work required to solve such problems will be very materially abreviated.

Thus, since the number of feet moved per minute is always $= \frac{\text{rate per hour in miles} \times 5280}{60} = \text{rate per hour in miles} \times \frac{5280}{60}$ $=$ rate per hour in miles $\times$ 88; therefore, whatever may be the rate, 88 is a constant multiplier.

Let $r =$ rate per hour in miles, then $88\,r =$ rate per min. in ft.
  $w =$ weight of train in tons, then $2000\,w =$ weight of train in lbs.
  $h =$ rise of the plane in every 100 feet.
  $f =$ friction per ton.
  $R =$ given atmospheric resistance at given speed, s.

Then units of work due per minute to friction $= fw \times 88\,r$.

" " " to gravity $= 2000w \times \frac{h}{100} \times 88\,r = 20\,hw \times 88\,r$.

Units of work due per min. to atmos. resist. $= R\left(\frac{r}{s}\right)^2 \times 88r$.

Units of work per min. in given H. P. = H. P. $\times$ 33000.

Hence H. P. $\times 33000 = fw \times 88r + R\left(\frac{r}{s}\right)^2 \times 88r \pm 20hw \times 88r$,

and factoring this, we get:

$$\text{H. P.} \times 33000 = \left(fw + R\left(\frac{r}{s}\right)^2 \pm 20hw\right) 88r.$$

$$\text{Therefore H. P.} = \left(fw + R\left(\frac{r}{s}\right)^2 \pm 20hw\right) \frac{88r}{33000}$$

$$\text{Or H. P.} = \left(fw + R\left(\frac{r}{s}\right)^2 \pm 20hw\right) \frac{r}{375} \quad \text{(I.)}$$

From this we obtain by transposition and reduction, and neglecting atmospheric resistance,

$$w = \frac{\text{H. P.} \times 375}{(f \pm 20h)r} \quad \text{(II.)}$$

$$r = \frac{\text{H. P.} \times 375}{(f \pm 20h)w} \quad \text{(III.)}$$

Since f is commonly $= 7$, $R. = 33$, and $s = 10$, these formulas become respectively,

$$\text{H. P.} = (7w + 33r^2 \pm 29\,hw) \frac{r}{375} \quad \text{(IV.}$$

$$w = \frac{\text{H. P.} \times 375}{(7 \pm 20h)r} \quad \text{(V.)}$$

$$r = \frac{\text{H. P.} \times 375}{(7 \pm 20h)w} \quad \text{(VI.)}$$

Example 174.—A train weighing 140 tons moves along a gradient having a rise of $\frac{1}{4}$ in 100 with the uniform speed of 30 miles per hour; taking friction and atmospheric resistance as usual, what is the H. P. of the locomotive; 1st when the train moves up the gradient? 2nd, when the train moves down the gradient?

SOLUTION.

Here $w = 140$, $r = 30$, $h = \frac{1}{4}$

$$\begin{aligned} \text{H. P.} &= (7\,w + \cdot 33r^2 \pm 20hw)\frac{r}{375} \\ &= (7 \times 140 + \cdot 33 \times 30^2 \pm 20 \times \tfrac{1}{4} \times 140)\tfrac{30}{375} \\ &= (980 + 297 \pm 700)\frac{2}{25} \\ &= \frac{1977 \times 2}{25} \text{ or } \frac{577 \times 2}{25} \\ &= 158 \cdot 16 \text{ or } 46 \cdot 16. \quad \textit{Ans.} \end{aligned}$$

Example 175.—A train drawn by a locomotive of 80 H. P. moves along an inclined plane having a rise of $\frac{1}{6}$ in 100 with a uniform velocity of 45 miles per hour; taking friction as usual and neglecting atmospheric resistance, what is the weight of the train?

SOLUTION.

Here H. P. $= 80$, $r = 45$, and $h = \frac{1}{6}$.

Then by formula (V.) $w = \dfrac{\text{H. P.} \times 375}{(7 \pm 20h)r} = \dfrac{80 \times 375}{(7 \pm 20 \times \frac{1}{6})45} = \dfrac{30000}{(7 \pm 3\frac{1}{3})\,45} =$ $\dfrac{30000}{10\frac{1}{3} \times 45}$ or $\dfrac{30000}{3\frac{2}{3} \times 45} = \dfrac{30000}{465}$ or $\dfrac{30000}{165} = 64 \cdot 51$ tons if the train is going up the gradient, or $181 \cdot 81$ tons if the train is going down the gradient.

For practice in the application of these formulas, work any of the foregoing problems.

THE MODULUS OF A MACHINE.

152. The modulus of a machine is the fraction which expresses the value of the work done compared with the work applied, the latter being expressed by unity.

Thus if $\frac{1}{7}$ of the work applied to a machine be lost in transmission, the modulus or useful work of that machine is $\frac{6}{7}$; if $\frac{3}{5}$ be lost in transmission, the modulus of the machine is $\frac{2}{5}$, &c.

153. The amount of work lost depends on friction, rigidity of cordage, &c., and in some machines is more than half of the whole work applied. The following table gives the moduli of machines for raising water:

TABLE OF MODULI.

MACHINE.	MODULUS.
Inclined chain pump,	$\frac{2}{5}$
Upright "	$\frac{1}{2}$
Bucket wheel,	$\frac{3}{5}$
Archimedian screw,	$\frac{7}{10}$
Pumps for draining mines,	$\frac{2}{3}$

EXAMPLE 176.—If 7 H. P. be applied to an upright chain pump, how many gallons of water will be raised per hour to the height of 50 feet ?

SOLUTION.

Work applied per hour $= 33000 \times 7 \times 60$.
Work done $= 33000 \times 7 \times 60 \times \frac{1}{2}$, since the modulus of the upright chain pump is $\frac{1}{2}$.
Work expended in raising 1 gallon of water 50 feet $= 10 \times 50$.

$$\therefore \text{Number of gallons} = \frac{33000 \times 7 \times 60 \times \frac{1}{2}}{10 \times 50} = 13860. \quad \textit{Ans.}$$

EXAMPLE 177.—What must be the H. P. of an engine to pump 9000 cubic feet of water per hour from a mine whose depth is 110 feet ?

SOLUTION.

Work of raising water per hour $= 9000 \times 62\frac{1}{2} \times 110$.
Effective work of one H. P. per hour $= 33000 \times 60 \times \frac{2}{3}$.

$$\therefore \text{H. P.} = \frac{9000 \times 62\frac{1}{2} \times 110}{33000 \times 60 \times \frac{2}{3}} = \frac{61875000}{1320000} = 46{\cdot}875. \quad \textit{Ans.}$$

WORK OF WATER.

154. When water falls from a height upon the float-boards of a wheel, &c., the quantity of work it performs is found by multiplying the weight of the water by the height through which it falls. (See Chap. VIII.)

STEAM ENGINES AND WORK OF STEAM.

155. A constant power is obtained from the confinement and regulated escape of steam in the various kinds of steam engines.

156. Steam engines, though differing very materially from one another in detail, are all modifications of two distinct machines, viz :—

1st. The high pressure steam engine, or non-condensing engine.

2nd. The low pressure steam engine, or condensing engine.

157. The high pressure engine, which is the simpler form of the two, consists essentially of a strong vessel or *boiler* in which the steam is generated, a *cylinder*, in which a tightly fitting *piston* moves backwards and forwards, an arrangement of *valves* so adjusted as to admit the steam alternately above and below the piston and also alternately open and close a way of escape into the air, and lastly various contrivances by which the oscillations of the piston may be converted into other kinds of motion suited to the work the engine is to perform.

158. In the low pressure engine, the space into which the steam drives the piston is converted, by means of a *condensing chamber*, into a vacuum, so that the motion of the piston is not resisted by atmospheric pressure, and steam generated at a low temperature can therefore be used.

159. The varieties of the low pressure engine are chiefly two,—the *single acting* and the *double acting engine.*

160. In the single acting engine the piston is driven forward by means of steam acting against a vacuum, and backward by the counterpoising weight of the machinery. The machine is therefore in action only half the time of the movement.

161. In the double acting engine the piston is driven both backward and forward by the steam acting against a vacuum on the opposite side, and the machine therefore acts continuously.

162. In the high pressure engine the piston moves both forwards and backwards against the pressure of the air.

163. The following are the leading ideas that enter into the construction and operation of the steam engine.

I. When steam is condensed, a vacuum is produced into which the adjacent bodies have a tendency to rush.

II. When cold water is placed in contact with steam, it condenses it with great rapidity, producing a vacuum; and this vacuum may be produced without cooling the cylinder containing the steam, if a communication be kept up between this and a vessel containing water.

III. The vapor of water exerts a considerable pressure even at comparatively low temperatures; for example, far below its boiling point.

IV. If the pressure exerted by the piston on a quantity of steam confined in a cylinder be less than the elastic force of the steam, the steam will expand and give motion to the piston.

V. If a vacuum be produced in a cylinder behind the piston, the atmospheric pressure will drive the piston backwards.

VI. The same quantity of fuel will convert the same quantity of water into steam whatever may be the pressure on its surface.

VII. The higher the pressure under which steam is generated, the smaller its bulk, and the greater its elastic force.

VIII. The same quantity of water converted into steam at any pressure will produce the same mechanical effect; i. e., if the pressure be low, the steam generated is large in quantity and possessed of comparatively little elastic force; if the pressure be high, the steam generated is of small quantity, but of high elastic force.

IX. One cubic inch of water converted into vapor produces 1696 cubic inches of steam, and, since the pressure of steam is, under ordinary circumstances, equal to that of the atmosphere, the mechanical force produced by the evaporation of one cubic inch of water is sufficient to raise 15 lbs. through 1696 inches or 141⅓ feet. This is the same, in effect, as raising 141⅓ times 15 lbs., i. e., 2120 lbs. through one foot. The conversion of one cubic inch of water into steam therefore does work equivalent to raising rather more than one ton weight through one foot. Deducting loss by friction and other causes, about 60 per cent. of this total force is available for use. One cubic foot of water evaporated in one hour will hence do work equal to about 60 per cent. of 1728 times 2120 units, or in other words about 2000000 units, which is about equivalent to the work of one horse for the same space of time.

A boiler then of 7, 8, 9, 10, &c., horse power is a boiler capable of evaporating 7, 8, 9, 10, &c., cubic feet of water per hour.

X. The common allowance of fuel for the steam engine is 10 lbs. of bituminous coals for every horse power of the boiler, (i. e., every cubic foot of water it evaporated per hour.) In Cornwall, however, this effect has been produced by the consumption of 5 lbs. of coal only. In the American boilers about 6½ lbs. of anthracite coal suffice for the evaporation of one cubic foot of water, or, in other words, the combustion of 1 lb. of coal is sufficient to evaporate 10 lbs. of water.

164. High pressure engines are commonly used where it is desirable to have the engine as simple, cheap, compact and light as possible, as the condensing apparatus renders the engine more costly and cumbrous. The high pressure engine is, however, far more liable to burst and get otherwise out of repair.

165. *The units of work performed per minute by a steam engine are found by multiplying together the pressure per square inch on the boiler, the area of the piston in inches, the length of the stroke of the piston in feet, and the number of strokes per minute.*

Thus let the pressure exerted on each square inch of the piston be 30 lbs., and let the piston make 40 strokes per minute of 3 feet each, also let the area of the piston be 100 square inches:

Now if a weight of 30 lbs. be placed on each square inch of the surface of the piston, the elastic force of the steam will be just sufficient to lift the loaded piston through the length of the stroke in opposition to gravity, then the work performed on 1 sq. in. of the piston would be 30×3 for each stroke.

Work performed on whole piston would be $30 \times 3 \times 100$ for each stroke.
Work " " " " $30 \times 3 \times 100 \times 40$ per minute.

166. In the high pressure engine, the pressure of the atmosphere, about 15 lbs. to the square inch, acts in opposition to the pressure of the steam; and in the low pressure or condensing engine a pressure of about 4 lbs. to the square inch of the piston is exerted by the vapor in the condensing chamber. Besides these, a resistance of 1 lb. per square inch is commonly allowed for the friction of the piston. Deducting these allowances from the total pressure, we obtain the effective pressure; and we must further make an allowance of $\frac{1}{7}$ of this for the friction of the whole engine.

Thus in the high pressure engine:

$$Load + \tfrac{1}{7}\ load + 1 + 15 = whole\ pressure.$$

In the condensing engine:

$$Load + \tfrac{1}{7}\ load + 1 + 4 = whole\ pressure.$$

For example,—if the whole pressure be 58 lbs. per square inch.

Then for the high pressure engine $58-1-15=42$ is the working pressure on the piston, and 42 is $\frac{8}{7}$ (i. e., load $+\frac{1}{7}$ load) of the useful pressure. and hence useful or effective pressure $= 42 \div \frac{8}{7} = 36\frac{3}{4}$.

For the low pressure engine $58-1-4=53$ = working pressure on the piston, and 53 is $\frac{8}{7}$ of the useful pressure. Therefore useful or effective pressure is $53 \div \frac{8}{7} = 46\frac{3}{8}$.

167. For finding the H. P. of a steam engine, let $p=$ useful pressure in lbs. on each square inch of the piston, $a=$ area of piston, $l=$ length of piston stroke in feet, and $n=$ number of strokes per minute.

$$\text{Then H. P.} = \frac{paln}{33000}. \quad \text{(I.)}$$

$$p = \frac{\text{H. P.} \times 33000}{aln}. \quad \text{(II.)}$$

$$a = \frac{\text{H. P.} \times 33000}{pln}. \quad \text{(III.)}$$

$$n = \frac{\text{H. P.} \times 33000}{pal}. \quad \text{(IV.)}$$

$$l = \frac{\text{H. P.} \times 33000}{pan}. \quad \text{(V.)}$$

EXAMPLE 178.—The piston of an engine has an area of 250 inches, and makes 110 strokes, of 5 feet each, per minute—taking the useful pressure of the steam as 28 lbs. per sq. inch, what is the H. P. of the engine?

SOLUTION.

Here $p = 28$, $a = 250$, $n = 110$, and $l = 5$.

Then (Formula I.) H. P. $= \frac{28 \times 250 \times 110 \times 5}{33000} = 116\frac{2}{3}$. *Ans.*

EXAMPLE 179.—The piston of a high pressure engine has an area of 1200 inches, and makes in each minute 30 strokes of 7 feet each—taking the gross pressure of the steam as 48 lbs. per square inch, what is the H. P. of the engine?

SOLUTION.

Here $48 = p + \frac{1}{7}p + 15 + 1$, or $\frac{8}{7}p = 32$, and hence $p = 32 \div \frac{8}{7} = 28$ lbs.

Then $p = 28$, $a = 1200$, $n = 30$, and $l = 7$.

By Formula I., H. P. $= \frac{28 \times 1200 \times 30 \times 7}{33000} = 213 \cdot 81$. *Ans.*

EXAMPLE 180.—The piston of a low pressure engine has a diameter of 20 in., and makes 60 strokes of 4 ft. each, per minute—the pressure of the steam on the boiler is 45 lbs. to the sq. inch, what is the H. P. of the engine?

SOLUTION.

Here $45 = p + \frac{1}{7}p + 4 + 1$, or $\frac{8}{7}p = 40$, and hence $p = 40 \div \frac{8}{7} = 35$.

$a^* = 10^2 \times 3 \cdot 1416 = 100 \times 3 \cdot 1416 = 314 \cdot 16$.

Then $p = 35$, $a = 314 \cdot 16$, $n = 60$, and $l = 4$.

H. P. $= \frac{35 \times 314 \cdot 16 \times 60 \times 4}{33000} = 79 \cdot 968$. *Ans.*

EXAMPLE 181.—In a steam engine of 32 horse power, the area of the piston is 500 inches, the length of the stroke 4 feet, and the useful pressure of the steam 33 lbs. to the sq. inch, how many strokes does the piston make per minute?

SOLUTION.

Here H. P. $= 32$, $a = 500$, $l = 4$, and $p = 33$.

Then (Formula IV.) $n = \frac{\text{H. P.} \times 33000}{pal} = \frac{32 \times 33000}{500 \times 4 \times 33} = 16$. *Ans.*

EXAMPLE 182.—In a low pressure steam engine of 190 H. P. the area of the piston is 1000 inches, the length of stroke 6 feet, and the number of strokes per minute 110, what is the useful pressure per square inch on the piston, and also what is the gross pressure of the steam?

* When the diameter of the piston is given, its area is found by multiplying the square of half the diameter by 3·1416.

SOLUTION.

Here H. P. = 190, $a = 1000$, $l = 6$, and $n = 100$.

Then (Formula II.) $p = \frac{190 \times 33000}{1000 \times 6 \times 110} = 9\frac{1}{2}$ lbs. = useful pressure.

And pressure on boiler [Art. 166] $= 9\frac{1}{2} + \frac{1}{7}$ of $9\frac{1}{2} + 4 + 1 = 15\frac{6}{7}$ lbs.

EXAMPLE 183.—In a high pressure engine the piston has an area of 800 inches, and makes 40 strokes per minute, of 10 feet each, what must be the pressure of the steam on the boiler in order that the engine may pump 120 cubic feet of water per minute from a mine whose depth is 400 feet—making the usual allowance for friction and the modulus of the pump ?

SOLUTION.

Here work done per minute $= 120 \times 62{\cdot}5 \times 400 = 3000000$ units.

Work applied, i. e., work of engine $= 3000000 \div \frac{2}{3} = 4500000$ units = H. P. $\times$ 33000.

Then by Formula II. $p = \frac{\text{H. P.} \times 33000}{aln} = \frac{4500000}{800 \times 10 \times 40} = 14\frac{1}{16}$ lbs. = useful pressure.

And Art. 166, gross pressure $= 14\frac{1}{16} + \frac{1}{7}$ of $14\frac{1}{16} + 15 + 1 = 32\frac{1}{14}$ lbs. *Ans.*

EXAMPLE 184.—The piston of a high pressure engine has an area of 600 inches, and makes 20 strokes per minute, each 8 feet in length, gross pressure of the steam 52 lbs. to the square inch. How many gallons of water per minute will this engine pump from a mine whose depth is 500 feet, making the usual allowance for friction and the modulus of the pump ?

SOLUTION.

Here $a = 600$, $l = 8$, $n = 20$, and since $52 = p + \frac{1}{7}p + 15 + 1$; $\frac{8}{7}p = 36$ and $p = 31\frac{1}{2}$.

Work of engine per minute $= paln = 31\frac{1}{2} \times 600 \times 8 \times 20 = 3024000$.

Useful work per minute $= 3024000 \times \frac{2}{3} = 2016000$.

Work of pumping 1 gallon of water to height of 500 feet $= 10 \times 500 = 5000$ units.

$\therefore$ No. of gallons pumped per minute $= \frac{2016000}{5000} = 403\frac{1}{5}$. *Ans.*

EXERCISE.

185. The piston of a low pressure steam engine is 40 inches in diameter and makes 40 strokes of 5 feet each per minute ;—the gross pressure of the steam is 37 lbs. per square inch what is the H. P. of the engine ? *Ans.* 213·248.

186. The piston of a high pressure engine is 20 inches in diameter and makes 50 strokes of 4 feet per minute ; taking the gross pressure of the steam as 40 lbs. per square inch and making the usual allowance for friction, what is the H. P. of the engine ? *Ans.* 39·984.

187. The piston of an engine has an area of 2400 inches and makes 16 strokes per minute, each 10 feet in length; the useful pressure of the steam on the piston is 20 lbs. per square inch, what is the H. P. of the engine?

Ans. 232·72.

188. In a high pressure engine of 140 H. P. the piston has an area of 1000 inches, and makes 20 strokes, of 5 feet each, per minute; what is the useful pressure of the steam on the piston and also the gross pressure per square inch?

Ans. Useful pressure = 46·2 lbs. per sq. in.
Gross pressure = 68·8 lbs. per sq. in.

189. In a low pressure engine of 100 H. P. the piston has an area of 200 inches, and makes 40 strokes per minute; the gross pressure of the steam is 45 lbs. per square inch. Required the length of the stroke made by the piston.

Ans. 11·785 feet.

190. In a high pressure engine of 80 H. P. the piston makes 44 strokes per minute, each 6 feet in length, and the gross pressure of the steam is 56 lbs. per square inch. What is the area of the piston? *Ans.* 285·714 sq. in.

191. How many cubic feet of water may be pumped per minute from a mine whose depth is 500 feet by an engine in which the piston has an area of 2000 inches, and makes 30 strokes per minute, each 8 feet in length, the useful pressure of the steam being 40 lbs. per square inch, and the usual allowance being made for the modulus of the pump?

Ans. 409·6 cubic feet.

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168. In all the modifications of the steam engine, the real source of work is the evaporating power of the boiler; the amount of work done by the engine depending not only upon the rapidity with which the water is evaporated, but also upon the temperature, and consequently the pressure under which the steam is produced. The following is a specimen of an experimental table, given by Pambour, showing the relation between the pressure, temperature, and volume of the steam produced by *one cubic foot* of water. By means of this table, we are enabled to ascertain the volume of the steam produced by a given quantity of water, when we know the pressure or temperature under which it is formed.

NOTE 1.—The first column gives the pressure in lbs. to the square inch under which the steam is produced; the second column shows the corresponding temperature, as indicated by Fahrenheit's thermometer; and the third column, the volume of the steam compared with the volume of the water which produced it. It will be observed that the lower the temperature, or what amounts to the same thing, the less the pressure under which the steam is formed, the greater its volume. Thus under the usual atmospheric pressure of 15 lbs. to the square inch [or at the common temperature of boiling water, 212° or 213° Fahr.], a cubic foot of water produces 1669 cubic feet of steam. If, however, the pressure be decreased to 1 lb. to the square inch, the steam is formed at the temperature of 103° Fahr., and occupies 20954 cubic feet; while if the pressure be increased to 30 lbs. to the square inch, the temperature required for the production of the steam rises to 251° Fahr., and the steam only occupies 882 cubic feet.

NOTE 2.—It has been shown by numerous experiments that the quantity of fuel requisite for the evaporation of a given quantity of water is invariably the same, no matter what may be the pressure under which the steam is produced. Hence it is obvious that it is most advantageous to employ steam of a high pressure.

TABLE

SHOWING THE VOLUME OF STEAM PRODUCED BY ONE CUBIC FOOT OF WATER AT THE CORRESPONDING PRESSURE AND TEMPERATURE.

PRESSURE to square inch.	TEMPERATURE, Fahrenheit's thermometer.	VOLUME of steam comp'd with that of the water producing it.	PRESSURE to square inch.	TEMPERATURE, Fahrenheit's thermometer.	VOLUME of steam comp'd with that of the water producing it.
1	103°	20954	55	288°	506
5	161°	4624	60	294°	467
10	192°	2427	65	299°	434
15	213°	1669	70	304°	406
20	228°	1280	75	309°	381
25	241°	1042	80	313°	359
30	251°	882	85	318°	340
35	260°	765	90	322°	323
40	268°	677	95	326°	307
45	276°	608	100	330°	293
50	282°	552	105	333°	281

169. If we let a = area of the piston in square inches.
l = length of stroke made by the piston.
n = number of strokes made per minute.
p = effective pressure to each square inch of the piston.
c = cubic feet of water evaporated per minute.
v = volume of one cubic foot of water in the form of steam under the given pressure p.

Th.n to find a, l, n, p, c, or v, when the others are given, we proceed as follows:

When p is given, v is found by the table.

Now the cubic feet of steam produced per minute $= cv$.

Cubic feet of steam used at each stroke of the piston $= \frac{al}{144}$*

$\therefore$ cubic feet of steam used in n strokes $= \frac{nal}{144} =$ also, the steam evaporated or used per minute.

Hence $\frac{nal}{144} = cv$. and from this by reduction we obtain

$$l = \frac{144cv}{na};\quad n = \frac{144cv}{al};\quad a = \frac{144cv}{nl};\quad c = \frac{nal}{144v}, \text{ and } v = \frac{nal}{144c}$$

When v is known p may be found by the table.

EXAMPLE 192.—The piston of a steam engine has an area of 200 square inches and makes a stroke of 4 feet in length, the boiler evaporating $\frac{3}{10}$ of a cubic foot of water per minute, under a pressure of 40 lbs. to the square inch. What number of strokes per minute does the piston make?

SOLUTION.

Here $a = 200$, $l = 4$, $c = \frac{3}{10} = \cdot 3$, and $p = 40$; also from table $v = 677$.

Then $n = \frac{144cv}{al} = \frac{144 + \cdot 3 + 667}{200 + 4} = 36{\cdot}558$ or $= 36\frac{1}{2}$. *Ans.*

EXAMPLE 193.—The piston of a steam engine has an area of 1000 inches, and makes 10 strokes per minute, each 3 feet in length, the boiler evaporates ·4 of a cubic foot of water per minute. What is the pressure under which the steam is generated?

SOLUTION.

Here $a = 1000$, $l = 3$, $n = 10$, and $c = \cdot 4$.

Then $v = \frac{nal}{144c} = \frac{10 \times 1000 \times 3}{144 \times \cdot 4} = 521$, whence by the table, p is between 50 and 55, or about 53 lbs.

EXAMPLE 194. The piston of a steam engine has an area of 80 inches, and makes 20 strokes per minute; the boiler evaporates $\frac{1}{10}$ of a cubic foot of water per minute under the pressure of 50 lbs. to the square inch. Required the length of the stroke made by the piston.

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* We divide by 144 because a, the area of the piston, is given in square inches, while l, the length of stroke, is given in feet. To find the cubic feet of steam we must multiply the length of stroke in feet by the area of the piston in square feet; i. e., by $\frac{a}{144}$

SOLUTION.

Here $a = 80$, $n = 20$, $c = \cdot 1$ and $p = 50$ and (table) $v = 552$.

Then $l = \frac{144cv}{na} = \frac{144 \times \cdot 1 \times 552}{20 \times 80} = 4.968$ ft. $= 4$ ft. $11\frac{1}{2}$ inches. *Ans.*

EXAMPLE 195.—The boiler of an engine evaporates $\frac{2}{5}$ of a cubic foot of water per minute under a pressure of 45 lbs. to the square inch; the piston has an area of 250 inches, and makes a stroke 4 feet in length. Required the number of strokes made by the piston per minute.

SOLUTION.

Here $a = 250$. $l = 4$, $c = \cdot 4$, $p = 45$, and hence (table) $v = 608$.

Then $n = \frac{144cv}{al} = \frac{144 \times \cdot 4 \times 608}{250 \times 4} = 35\cdot 0208$, i.e. 35 strokes per minute. *Ans.*

EXERCISE.

196. The boiler of a steam engine evaporates $\frac{4}{5}$ of a cubic foot of water per minute under a pressure of 65 lbs. to the square inch. If the piston has an area of 144 square inches, and makes strokes 5 feet in length, how many strokes are made per minute? *Ans.* 69·44

197. The piston of an engine has an area of 288 inches, and makes 7 strokes per minute. If the boiler evaporates $\frac{7}{10}$ of a cubic foot of water per minute under a pressure of 55 lbs. to the square inch, what is the length of the stroke of the piston? *Ans.* $25\frac{3}{10}$ feet.

198. The piston of an engine makes 10 strokes of 6 feet each per minute; the boiler evaporating $\frac{1}{2}$ a cubic foot of water per minute under a pressure of 25 lbs. to the square inch, what is the area of the piston? *Ans.* 1250·4 inches.

199. In a steam engine the piston having an area of 720 inches makes 20 strokes, of 3 feet each, per minute, what volume of water converted into steam under a pressure of 20 lbs. to the square inch, is evaporated per minute by the boiler? *Ans.* $\frac{15}{64}$ of a cubic foot.

200. The piston of a steam engine has an area of 600 inches, and makes 12 strokes, of 10 feet each, per minute. Now if the boiler evaporates 1 cubic foot of water per minute, what is the volume of the steam produced per minute at the pressure under which it is generated?

Ans. Volume = 500 cubic feet.
Pressure = nearly 55 lbs. to the square inch.

170. To find the useful H. P. of an engine when a, n, l, c, and v are given, we proceed as follows:

Find the pressure per square inch of the steam from the Table, and thence Art. 166 the useful load on each square inch of the piston; find also when required any of the other quantities, a, n, or l, and then apply the rules given in Art. 167.

EXAMPLE 201.—What is the useful load per square inch on the piston, and what is the effective horse power of a high pressure engine in which the area of the piston is 200 inches, the length of stroke 6 feet, the effective evaporation of the boiler $\frac{2}{5}$ of a cubic foot per minute, and the pressure of the steam 70 lbs. to the square inch?

SOLUTION.

By Art. 166, $70 = \frac{8}{7}p + 15 + 1$, and hence $p = 54 \div \frac{8}{7} = 47{\cdot}25 =$ useful oad.

By Art. 169, $n = \frac{144\,cv}{al} = \frac{144 \times {\cdot}4 \times 406}{200 \times 6} = 19{\cdot}488$.

Hence we have $n = 19{\cdot}488$, $p = 47{\cdot}25$, $a = 200$, $l = 6$.

Then Art. 167, H. P. $= \frac{paln}{33000} = \frac{47{\cdot}25 \times 200 \times 6 \times 19{\cdot}488}{33000} = 33{\cdot}48$. *Ans.*

EXAMPLE 202.—What is the effective horse power of a low pressure engine in which the piston has an area of 288 inches and makes every minute 16 strokes, the boiler converting $\frac{1}{2}$ of a cubic foot of water per minute into 304 cubic feet of steam?

SOLUTION.

Since $\frac{1}{2}$ of a cubic foot of water produces 304 cubic feet of steam, 1 cubic foot of water would produce 608 cubic feet of steam, and hence [Table) the gross pressure of the steam is 45 lbs. to the square inch.

Then (Art. 166) $45 = \frac{8}{7}p + 4 + 1$, or $\frac{8}{7}p = 40$ whence $p = 35$.

Also (Art. 169) $l = \frac{144\,cv}{na} = \frac{144 \times {\cdot}5 \times 608}{288 \times 16} = 9\frac{1}{2}$ ft.

Then $a = 288$, $l = 9\frac{1}{2}$, $n = 16$, and $p = 35$.

Hence Formula I, Art. 167, H, P. $= \frac{paln}{33000} = \frac{35 \times 288 \times 9\frac{1}{2} \times 16}{33000} = 46{\cdot}429$. *Ans.*

EXERCISE.

203. What is the effective horse power of a high pressure engine in which the piston has an area of 360 inches and makes 20 strokes per minute,—the boiler evaporating $\frac{3}{4}$ of a cubic foot of water per minute under a pressure of 40 lbs. to the square inch? *Ans.* H. P. $= 46{\cdot}528$

204. The piston of a low pressure steam engine has an area of 432 inches, and makes strokes 10 feet in length. Now, if the boiler evaporates ·9 of a cubic foot of water per minute under a pressure of 25 lbs. to the square inch, what is the useful H. P. of the engine? *Ans.* H. P. $= 71{\cdot}613$.

205. In a high pressure engine the area of the piston is 600 inches, the length of stroke is 6 feet, the effective evaporation of the boiler is $\frac{3}{8}$ of a cubic foot per minute, and the pressure of the steam in the cylinder 80 lbs. to the square inch. Required the H. P. *Ans.* H. P. $= 32{\cdot}897$.

CHAPTER V.

HYDROSTATICS.

171. Fluidity consists in the transmission of pressure in all directions, or, a fluid may be defined to be a body whose particles are so free to move among one another that they yield to any pressure, however small, that may be applied to them.

172. The term fluid is commonly applied to bodies in both the liquid and gaseous state.

173. Fluids are divided into two classes:

1st. Elastic fluids, of which atmospheric air is the type.

2nd. Non-elastic fluids, of which water is the representative.

NOTE.—Water was formerly thought to be absolutely incompressible, but recent experiments show that water is diminished in volume $\frac{1}{22000}$ of its bulk for each atmosphere of pressure upon it; or in other words a pressure of 2000 atmospheres or 30000 lbs. to the square inch would compress 11 cubic feet into 10 cubic feet. Alcohol is about twice as compressible as water.

174. Liquids, by which term we mean non-elastic fluids, differ from gases principally in having less elasticity and compressibility.

175. Liquids differ from solids chiefly in the fact that their particles are less under the influence of the attraction of cohesion, and therefore have a freer motion among themselves, in consequence of which each atom is drawn separately towards the earth by the force of gravity; hence:—

I. A liquid, confined in any vessel, presses equally in all directions—upwards, downwards, and laterally.

II. The surface of a liquid in a state of rest is always level.

III. A liquid rises to the same height in all the tubes connected with a common reservoir, whatever may be their form or capacity.

NOTE.—The fact that a liquid exerts a downward pressure is self-evident and requires no illustration.

The lateral pressure of a liquid is shown by its spouting from holes pierced in the side of the vessel in which it is contained.

The upward pressure is shown by taking a glass cylinder, open at both ends, and having one end accurately ground. A plate of ground glass is held to this end by means of a piece of string passing through the cylinder and the closed end of the instrument then immersed in water to a small depth. Upon letting go the string the plate is still held against the cylinder by the upward pressure of the water; it will even sustain any weight, which, together with the plate itself, is not greater than the weight of the water that would enter the cylinder if the plate were removed.

176. When two liquids of different densities are placed in the opposite branches of an inverted syphon or bent tube—their heights in the two legs above the point of contact will be inversely as their densities.

NOTE.—This may easily be proved by placing mercury and water in a bent graduated glass tube, when it will be found that the column of water will be 13½ times as high as the column of mercury since the latter is about 13½ times as heavy as the former.

177. The amount of downward pressure exerted by a liquid in any vessel is equal to that of a column of the same liquid, whose base is equal to the area of the bottom of the vessel, and whose height is equal to the depth of the liquid, whatever may be the form or capacity of the vessel.

NOTE 1.—To illustrate this fact we procure three vessels, having bottoms of the same area, and sides, in the first perpendicular, in the second converging towards the top, and in the third diverging towards the top. The bottoms are hinged and are held in their places by a cord passing over a pulley and terminating in a scale pan in which are placed weights to a certain amount. Water is then carefully poured into the vessel having the perpendicular sides until its downward pressure is just sufficient to force out the bottom when its depth is accurately measured. Upon using either of the other vessels it is found that the bottom remains fixed until the water reaches this depth and is then forced open. This arises from the fact that when the sides are perpendicular the bottom supports the whole weight of the water; when the vessel is wider at top than at bottom a portion of the downward pressure is sustained by the sides, while, when the vessel is wider at the bottom than at top, the particles near the bottom are pressed upon by the whole column of liquid above them and their downward and lateral pressure is the same as it would be were the column of liquid of the same dimensions throughout as the base of the vessel.

NOTE 2.—Care should be taken not to confound weight with pressure, inasmuch as the weight is in proportion to the quantity of liquid, but the pressure is in proportion to the extent of base and the perpendicular height of the liquid. For example, the weight of the water contained in a conical vessel is found by multiplying the area of the base by one-third of the perpendicular height; but the pressure, by multiplying the area of the base by the whole height. It follows that in a conical vessel the downward pressure is equal to three times the weight of the liquid. Hence in a vessel with perpendicular sides, the pressure equals the weight; if the sides diverge upwards, the pressure is less than the weight; and if the sides converge upwards, the pressure is greater than the weight.

178. A cubic inch of water of the temperature of 60° Fahr. weighs 0·03616 lbs. Avoir., a cubic foot at the same temperature weighs 1000 ounces or 62·5 lbs., and a gallon, 10 lbs.

179. The pressure of a liquid on a *vertical or inclined* surface is equal to the weight of a column of the same liquid whose base is equal to the area of the surface pressed, and height equal to the depth of the centre of gravity of the pressing liquid beneath its level surface.

Or, more simply, the lateral pressure exerted by any liquid on the side of a vessel is found in lbs. by multiplying the area of the surface pressed by half the depth of the liquid, and this product by the weight in lbs. of one cubic foot of that liquid.

NOTE.—It follows that in a cubical vessel filled with any liquid the pressure on the side is equal to half the weight of the liquid, and hence the whole pressure exerted by the liquid, downward and laterally, is equal to three times the weight of the liquid.

APPLICATION OF THE PRINCIPLES CONTAINED IN ARTS. 176-179.

EXAMPLE 206.—What downward pressure is exerted on the bottom of an upright cylindrical vessel having a diameter of 20 feet—the water filling it to the depth of 12 feet?

SOLUTION.

Here, since the sides are perpendicular, the downward pressure = the weight.

Area of the bottom $= 10^2 \times 3{\cdot}1416 = 100 \times 3{\cdot}1416 = 314{\cdot}16$ feet.

Cubic feet of water $= 314{\cdot}16 \times 12 = 3769{\cdot}92$.

$\therefore$ Weight $= 3769{\cdot}92 \times 62{\cdot}5 = 235620$ lbs. = pressure. *Ans.*

EXAMPLE 207.—If olive oil and milk be placed in the two legs of a bent tube or inverted syphon, when the height of the column of milk above the point of junction is 20 inches, what will be the height of the column of oil?

SOLUTION.

From the table of specific gravities Art. 198, the weight of milk is to that of olive oil as 1030 : 915.

Hence (Art. 176) $915 : 1030 :: 20 : \frac{1030 \times 20}{915} = 22\frac{1}{2}$ inches. *Ans.*

EXAMPLE 208.—If mercury and ether are placed in a bent tube, as in the last example, what will be the height of the column of mercury when that of the ether is 100 inches high?

SOLUTION.

From the table of specific gravities the weight of mercury is to that of ether as 13596 : 715.

Hence (Art. 176) $13596 : 715 :: 100 : \frac{715 \times 100}{13596} = 5\frac{1}{4}$ inches. *Ans.*

EXAMPLE 209.—What will be the lateral pressure exerted against the side of a cistern,—the side being 20 feet long and the water 12 feet deep?

SOLUTION.

Area of the surface pressed $= 20 \times 12 = 240$ feet.
Then (Art. 178) lateral pressure = area multiplied by half the depth $\times$ $62{\cdot}5 = 240 \times 6 \times 62{\cdot}5 = 90000$ lbs. *Ans.*

EXAMPLE 210.—What is the amount of the pressure exerted against one side of an upright gate of a canal, the gate being 27 feet wide and the water rising on the gate to the height of 8 feet?

SOLUTION.

Area of the gate $= 27 \times 8 = 216$ feet, and half the depth of the water $= 4$ ft.
Then (Art. 179) pressure $= 216 \times 4 \times 62{\cdot}5 = 54000$ lbs. *Ans.*

EXAMPLE 211.—What is the amount of pressure exerted against a mill-dam whose length is 220 feet, the part submerged being 9 feet wide, and the water being 7 feet deep?

SOLUTION.

Area of part submerged $= 220 \times 9 = 1980$ feet, and half the depth of water $= 3{\cdot}5$ feet.
Then (Art. 179) pressure $= 1980 \times 3{\cdot}5 \times 62{\cdot}5 = 433125$ lbs. *Ans.*

EXAMPLE 212.—If the body of a fish have a surface of 5 square feet, what will be the aggregate pressure it sustains at the depth of 100 feet?

SOLUTION.

In this and similar examples the body of the fish has to sustain a pressure equal to the weight of a column of the water having a base equal in area to the surface of the fish and a height equal to the depth of the fish beneath the surface of the water.
Then volume of water sustained by the body of the fish $= 5 \times 100 = 500$ cubic feet.
Hence pressure $= 500 \times 62{\cdot}5 = 31250$ lbs. *Ans.**

EXAMPLE 213.—If a man whose body has a surface of 15 square feet dives in water to the depth of 70 feet, what pressure does his body sustain?

SOLUTION.

Column of water sustained by man's body at depth of 70 feet $= 15 \times 70 = 1050$ cubic feet.
Hence pressure $= 1050 \times 62{\cdot}5 = 65625$ lbs. *Ans.*

EXAMPLE 214.—To what depth may an empty closed glass vessel just capable of sustaining a pressure 170 lbs. to the square inch be sunk in water before it breaks?

SOLUTION.

From Art. 178 we find that a cubic inch of water at the common temperature of 60° Fahr. weighs 0.03616 of a pound Avoirdupois.
Hence the vessel may be sunk as many inches as .03616 lbs. is contained times in 170 lbs.
That is depth $= 170 \div 0.03616 = 4701\frac{1}{3}$ inches $= 391$ feet $9\frac{1}{3}$ inches. *Ans.*

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* In this and following examples involving the same principle, we make no allowance for the increased pressure at great depths.

EXAMPLE 215.—If an empty corked bottle be sunk to the depth of 130 feet before the cork is driven in,—what pressure to the square inch was the cork capable of sustaining before entering the bottle?

SOLUTION.

Column of water sustained by each square inch of the cork $= 130 \times 12 = 1560$ cubic inches.

Then weight sustained by each square inch of the cork $= 1560 \times 0.03616 = 56{\cdot}4$ lbs. *Ans.*

EXERCISE.

216. What is the amount of pressure exerted against one side of the upright gate of a canal,—the gate being 24 feet wide and submerged to the depth of 10 feet?
Ans. 75000 lbs.

217. What is the amount of pressure exerted against a mill-dam,—the part submerged being 10 feet wide and 80 feet long and the depth of the water being 8 feet?
Ans. 200000 lbs.

218. What is the pressure sustained by the sides of a cubical water tight box placed in water at the depth of 120 feet beneath the surface,—each edge of the box being 5 feet long? *Ans.* 1125000 lbs.

219. At what depth beneath the surface will a closed glass vessel, capable of sustaining a pressure of 79 lbs. to the square inch, break? *Ans.* 182 ft. $0\frac{3}{4}$ inch.

220. What pressure is sustained by the body of a man at the depth of 30 feet,—assuming that his body has a surface of $1\frac{1}{2}$ square yards? *Ans.* $25312\frac{1}{2}$ lbs.

221. What is the amount of pressure exerted against one side of the upright gate of a canal,—the gate being 30 feet wide and submerged to the depth of 5 feet?
Ans. $23437\frac{1}{2}$ lbs.

222. In a glass tube bent in the form of a syphon a column of turpentine is balanced by means of a column of sea water,—if the height of the former be 20, 30, or 47 inches what in each case will be the height of the latter?
Ans. $16\frac{9}{16}$, $25\frac{3}{5}$ or $39\frac{1}{6}$ inches.

223. What is the downward pressure, the pressure on each side and also the pressure on each end of a rectangular cistern,—14 feet long, and 9 feet wide—the water being 10 feet deep? *Ans.* Downward pressure $=$ 78750 lbs.
Pressure on side $=$ 43750 lbs.
Pressure on end $=$ 28125 lbs.

224. What amount of pressure is sustained by the body of a whale the depth of 260 feet, upon the supposition that his body presents a surface of 200 square yards?
Ans. 29250000 lbs.

225. In a glass tube bent in the form of a syphon a column of mercury is balanced in succession by a column of alcohol and a column of sulphuric acid. If the height of the former be 10 inches, what in each case will be the height of the latter? *Ans.* Alcohol = $171\frac{3}{8}$ inches.
Sulphuric acid = $73\frac{4}{5}$ inches.

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180. To find the pressure exerted against a vertical or inclined surface at some given depth beneath the surface of the water:

RULE.

Add the depth of the upper part of the surface to that of the lower part, and divide the sum by 2. The result is the mean height of the columns of water pressing on that surface.

Then multiply the area of the surface by the mean height of the water pressing it, and the result by the weight in lbs., of one cubic foot of water.

EXAMPLE 226.—What amount of pressure is sustained by one square yard of the side of a canal, the upper edge being 10 feet and the lower edge 12 feet beneath the surface of the water?

SOLUTION.

Mean weight of column of water pressing the given surface $= \frac{10+12}{2} =$ 11 feet, and area of surface = 9 square feet.

Then pressure $= 9 \times 11 = 99 \times 62{\cdot}5 = 6187\frac{1}{2}$ lbs. *Ans.*

EXAMPLE. 227.—An upright flood gate is so placed in a canal, that the water is just level with the top of the gate.—Assuming the gate to be 30 feet long and 20 feet wide, what pressure is sustained by the lower half of one side?

SOLUTION.

The upper edge of the half to which the problem refers is 10 feet beneath the surface, and the lower edge 20 feet, therefore the mean height of the column of water pressing against it is $\frac{10+20}{2} = 15$ feet.

Also area of part of gate given $= 30 \times 10 = 300$ square feet.

Hence pressure $= 300 \times 15 \times 62{\cdot}5 = 281250$ lbs. *Ans.*

181. In problems similar to the last a better rule to use may be derived from the following consideration:

The pressure on the whole gate is to the pressure on any fraction of it measured from the top, in the duplicate ratio of 1 to that fraction.

Hence to find the pressure on any part of the gate we have the following:

RULE.

FIRST.—If the part of the gate be measured from the top downwards.

Find the pressure on the whole gate by Art. 179, *and multiply it by the square of the given fraction.*

SECOND.—If the part of the gate be measured from the bottom upwards.

Take the given fraction from 1, *square the remainder, and subtract it from unity.*

Multiply the pressure on the whole gate by the fraction thus obtained, and the result will be the pressure on the given fraction.

EXAMPLE 228.—The flood-gate of a canal is 16 feet wide and 12 feet deep, and is placed vertically in the canal, the water being on one side only and just level with the upper edge of the gate; Required the pressure—1st. On the whole gate.
2nd. On the upper third of the gate.
3rd. On the lower half of the gate.
4th. On the upper two-fifths of the gate.
5th. On the lower two-elevenths of the gate.

SOLUTION.

I. Pressure on the whole gate $= 16 \times 12 \times 6 \times 62{\cdot}5 = 72000$ lbs.

II. Pressure on upper third $=$ whole pressure $\times (\frac{1}{3})^2 = 72000 \times \frac{1}{9} =$ 8000 lbs.

III. Pressure on lower half $=$ whole pressure $\times \left\{1-(\frac{1}{2})^2\right\} = 72000 \times \frac{3}{4} =$ 54000 lbs.

IV. Pressure on upper two-fifths $=$ whole pressure $\times (\frac{2}{5})^2 = 72000 \times \frac{4}{25}$ $= 11520$ lbs.

V. Pressure on lower two-elevenths $=$ whole pressure $\times \left\{1-(\frac{9}{11})^2\right\} =$ $72000 \times \frac{40}{121} = 23801{\cdot}6528$ lbs.

In III we take the given fraction $\frac{1}{2}$ from unity, this leaves $\frac{1}{2}$ which we square and again subtract from unity and thus obtain $\frac{3}{4}$ for the multiplier.

In V we take the given fraction $\frac{2}{11}$ from unity, this gives us $\frac{9}{11}$ which we square and again snbtract from unity thus obtaining $\frac{40}{121}$ for the multiplier.

EXAMPLE 229.—If a flood-gate be placed as in last example what pressure will be exerted on the upper $\frac{3}{7}$, and what on the lower $\frac{2}{5}$ of the gate if it be 10 feet wide and 12 feet deep?

SOLUTION.

We first find the pressure on the whole gate by Art. 179.

Then for the upper $\frac{3}{7}$ we multiply the whole pressure by the square of $\frac{3}{7}$.

For the lower $\frac{2}{5}$ we subtract $\frac{2}{5}$ from 1, this gives us $\frac{3}{5}$ which we square and thus obtain $\frac{9}{25}$, then we subtract $\frac{9}{25}$ from 1 and thus obtain $\frac{16}{25}$, lastly we multiply the whole pressure by this $\frac{16}{25}$.

Whole pressure $= 10 \times 12 \times 6 \times 62{\cdot}5 = 45000$ lbs.
Pressure on upper $\frac{3}{7} = 45000 \times \frac{9}{49} = 8265\frac{1}{3}$ lbs.
Pressure on lower $\frac{2}{5} = 45000 \times \frac{16}{25} = 28800$ lbs.

EXERCISE.

230. The flood-gate of a canal is 30 feet wide and 10 feet deep, and is placed vertically in the canal, the water being on one side only and level with the top, required the pressure—1st.

On the whole gate, 2nd. On the upper half of the gate; 3rd. On the lower half of the gate; 4th. On the lowest two-sevenths of the gate.

Ans. Pressure on whole gate $= 93750$ lbs.
" upper half $= 23437\frac{1}{2}$ "
" lower half $= 70312\frac{1}{2}$ "
" lowest two-sevenths $= 45918\frac{18}{49}$ "

231. A hollow globe has a surface of 7 square feet, and is sunk in water to the depth of 150 feet. Required the total pressure it then sustains. *Ans.* 65625 lbs.

232. What pressure is exerted against one square yard of an embankment if the upper edge of the square yard be 11 ft. and the lower edge 13 feet beneath the surface of the water? *Ans.* 6750.

233. A hollow glass globe is sunk in water to the depth of 400 feet, at which point it breaks. Required the extreme pressure to the square inch which the vessel was capable of sustaining. *Ans.* 173·568 lbs.

234. Required the pressure sustained by the body of a man at a depth of 100 yards beneath the surface of water—assuming the man's body to have a surface of 15 square feet.
Ans. 281250 lbs.

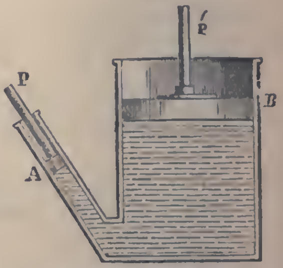
235. A flood-gate 16 feet long is submerged to the depth of 9 feet in water; what pressure is exerted against each side of it? *Ans.* 40500 lbs.

236. A mill-dam is 120 feet long and 11 wide, the water being exactly level with the top of the dam and the lower edge of the dam 7 feet beneath the surface. 1st. What will be the pressure exerted against the whole dam. 2nd. What pressure will be exerted against the upper part of the dam. 3rd. What pressure will be exerted against the lower half of the dam? *Ans.* Against whole dam 288750 lbs.
" upper half $72187\frac{1}{2}$ lbs.
" lower half $216562\frac{1}{2}$ lbs.

237. A flood-gate 26 feet wide is submerged perpendiculary to the depth of 12 feet; find 1st. The pressure against one side of the whole part submerged. 2nd. The pressure against the lower half. 3rd. The pressure against the lowest third. 4th. The pressure against the lowest sixth.
Ans. 117000 lbs. whole gate.
87750 lbs. lower half.
65000 lbs. lowest third.
35750 lbs. lowest sixth.

182. If water be confined in a vessel and a pressure to any amount be exerted upon any one square inch of the surface of that water, a pressure to an equal amount will be transmitted to every square inch of the interior surface of the vessel in which the water is confined.

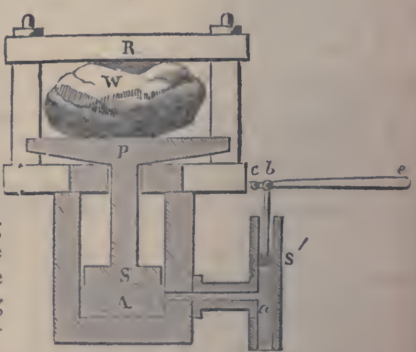
Fig. 16.



NOTE.—In the accompanying figure suppose the piston P has an area of 1 square inch, and the piston p' an area of 100 square inches, then if 1 lb. pressure be applied to P a weight of 100 lbs. must be applied to p' in order to maintain equilibrium. It is this property of equal and instant transmission of pressure which enables us to make use of hydrostatic pressure as a mechanical power, and it is upon this principle that Bramah's Hydrostatic Press is constructed.

183. Bramah's Hydrostatic Press consists of two strong metalic cylinders A and a, one many times as large as the other, connected together by a tube. The small cylinder is supplied with a strong forcing pump s', and the larger one with a tightly fitting piston S, attached to a firm platform or strong head P. Both the cylinders and the communicating tube contain water, and when downward pressure is applied to the water in the smaller cylinder, by means of the attached forcing pump, the piston in the larger is forced upward by a pressure as much greater than the downward pressure in the smaller, as the section area of the larger cylinder is greater than that of the sm

Fig. 17.



For example, if the smaller cylinder have an area of half a square inch, and the large cylinder an area of 500 square inches, then the upward pressure in the latter will be 1000 times as great as the downward pressure in the former.

184. Bramah's Hydrostatic Press is used for pressing paper, cotton, cloth, gunpowder. and other things—also for testing the strength of rop.., for uprooting trees, and for other purposes.

185. To find the relation between the force applied and the pressure obtained in Bramah's Hydrostatic Press.

RULE.

I. If the power be applied by means of a lever, find the amount of downward pressure in the smaller cylinder by the rule in Art. 77.

II. Divide the sectional area of the larger cylinder by that of the smaller cylinder, and multiply the quotient by the power applied to the smaller cylinder.

EXAMPLE 238.—In a hydrostatic press the force pump has a sectional area of one square inch; the large cylinder a sectional area of one square foot, the force pump is worked by means of a lever whose arms are to one another as 21 : 2. If a power of 20 lbs. be applied to the extremity of the lever what will be the upward pressure exerted against the piston in the large cylinder?

SOLUTION.

Power applied to a force pump $= \frac{20 \times 21}{2} = 21$ lbs.

Sectional area of smaller cylinder = 1 inch, and of a larger cylinder = 144 inches.

Then $144 \div 1 = 144 \times 210 = 30240$ lbs. *Ans.*

EXAMPLE 239.—In a hydrostatic press the sectional areas of the cylinders are $\frac{1}{3}$ of an inch and 150 inches, and the power lever is so divided that its arms are to one another as 7 to 43. What pressure will be exerted by a power of 100 lbs. applied at the extremity of the long arm of the lever?

SOLUTION.

Downward pressure in small cylinder $= \frac{100 \times 43}{7} = 614\frac{2}{7}$ lbs.

Upward pressure in large cylinder $= \frac{150}{\frac{1}{3}} \times 614\frac{2}{7} = 450 \times 614\frac{2}{7} = 276428\frac{4}{7}$ lbs. *Ans.*

EXAMPLE 240.—The area of the small piston of a hydrostatic press is $\frac{1}{2}$ an inch and that of the larger one 300 inches, the lever is 30 inches long and the piston rod is placed 5 inches from the fulcrum (so as to form a lever of the second order) what power must be applied to the end of the lever in order to produce an upward pressure in the cylinder of 1000000 lbs.?

SOLUTION.

Downward pressure in smaller cylinder $= 1000000$ lbs. $\div \frac{300}{\frac{1}{2}} = 1000000$ lbs. $\div 600 = 1666\frac{2}{3}$ lbs.

Then power applied $= 1666\frac{2}{3}$ lbs. $\div \frac{30}{5} = 1666\frac{2}{3} \div 6 = 277\frac{7}{9}$ lbs. *Ans.*

EXERCISE.

241. In a hydrostatic press the area of the small cylinder is one inch, and that of the large one 300 inches, the force pump is worked by a lever of the second order 30 inches long, having the piston rod 2 inches from the fulcrum; if a pressure of 50 lbs. be applied to the lever, what upward pressure will be produced in the large cylinder?

Ans. 225000 lbs.

242. In a hydrostatic press the force pump has a sectional area of half an inch, the large cylinder a sectional area of 200 inches; the force pump is worked by means of a lever whose arms are to one another as 1 to 50; now suppose a force of 50 lbs. be applied to the extremity of the lever, what will be the upward pressure exerted against the piston in the large cylinder?

Ans. 1000000.

243. In a hydrostatic press the small cylinder has an area of one inch, and the large one an area of 500 inches, the pump lever is so divided that its arms are to one another as 1 to 25. What will be the upward pressure against the piston in the large cylinder produced by a force of 100 lbs. acting at the extremity of the lever?

Ans. 1250000.

244. The area of the small piston of a hydrostatic press is $\frac{3}{4}$ of an inch, and that of the large one 120 inches—the arms of the lever by which the force pump is worked are to one another as 40 to 3. Required the upward pressure exerted against the piston of the large cylinder by a power of 17 lbs. applied at the extremity of the lever?

Ans. $36266\frac{2}{3}$ lbs.

245. The area of the small piston of a hydrostatic press is $1\frac{1}{2}$ inch, and that of the large one 200 inches—the arms of the lever by which the force pump is worked are to one another as 20 to $1\frac{1}{2}$. What power applied at the extremity of the lever will produce a pressure of 750000 lbs.?

Ans. $421\frac{7}{8}$ lbs.

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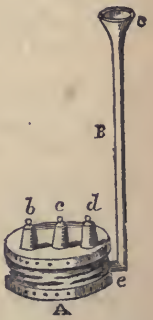
186. Since the pressure of water upon a given base depends upon the height of the liquid and not upon its quantity, it follows that:—

Any quantity of water, however small, may be made to balance the pressure of any other quantity however great, or to raise any weight however large.

NOTE.—This is what is commonly called the *Hydrostatic Paradox.* In reality, however, there is nothing at all paradoxical in it; since, although a pound of water may be made to balance 10 lbs., or 1000 lbs., or 100,000 lbs., it does it upon precisely the same principle that the power balances the weight in the lever and other mechanical powers. Thus in order to raise 20 lbs. of water by the descending force of 1 lb., the latter must descend 20 inches in order to raise the former 1 inch. Hence what is called the hydrostatic paradox is in strict conformity to the principle of virtual velocities.

187. This principle is illustrated by an instrument called the Hydrostatic Bellows, which consists of a pair of boards united together by leather, as in the common bellows, and made water-tight. From the upper board there rises a long tube *B*, finished with a funnel-shaped termination, *C*.

Fig. 18.



NOTE. When water is poured into the tube an upward pressure is exerted against the upper board as much greater than the weight of the water in the tube as the area of the board is greater than the sectional area of the tube.

For example, if the sectional area of the tube be ¼ of an inch, and the area of the board be 250 in., then the area of the board will be 1000 times as great as that of the tube, and consequently 1 lb. of water in the tube will exert a pressure of 1000 lbs. against the upper board of the bellows.

188. To find the upward pressure exerted against the board of a hydrostatic bellows by the water contained in the tube.

RULE.

Divide the sectional area of the board by that of the tube, and multiply the result by the weight of the water in the tube.

NOTE.—*The weight of water in the tube is found by multiplying the sectional area of the tube by the height of the water in inches and the product, which is cubic inches of water, by* 0·03616 *lbs., the weight of one cubic inch of water.*

EXAMPLE 246.—The upper board of a hydrostatic bellows has an area of 1 foot, the tube has a sectional area of ½ an inch and is filled with water to the height of 7 feet. What upward pressure is exerted against the top board of the bellows?

SOLUTION.

Cubic inches of water in the tube $= \frac{1}{2} \times 84 = 42$.
Weight of water in tube $= 0{\cdot}03616 \times 42 = 1{\cdot}51872$.

Upward pressure against bellows board $= 1{\cdot}51872 \times \frac{144}{\frac{1}{2}} = 1{\cdot}51872 \times 288$

$= 437{\cdot}39$ lbs Ans.

EXAMPLE 247.—In a hydrostatic bellows the board has an area of 200 inches and the tube a sectional area of $\frac{1}{4}$ of an inch. What upward pressure is exerted on the board by 7 lbs. of water in the tube.

SOLUTION.

Upward pressure $= 7 \times \frac{200}{\frac{1}{4}} = 7 \times 800 = 5600$ lbs. *Ans.*

EXERCISE.

248. In a hydrostatic bellows the board has an area of 250 inches, the tube has a sectional area of $1\frac{1}{4}$ inch, and contains 11 lbs. of water. What is the amount of upward pressure exerted against the board of the bellows?

Ans. 2200 lbs.

249. The board of a hydrostatic bellows has an area of 300 inches, the tube has a sectional area of 1 inch, and is filled with water to the height of 10 feet—what pressure will be exerted against the upper board of the bellows?

Ans. 1301·76 lbs.

250. The tube of a hydrostatic bellows has a sectional area of ·72 of an inch, and is filled with water to the height of 50 feet—what weight will be sustained on the bellows' board if the latter have an area of 3 feet?

Ans. 9372·672 lbs.

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189. A body immersed in any liquid will either float, sink, or rest in equilibrium, according as it is specifically lighter, heavier, or the same as the liquid.

190. A floating body displaces a quantity of liquid equal to its own weight.

191. A body immersed in any liquid loses a portion of its weight equal to the weight of the liquid displaced, and, hence by weighing a body first in air and then in water, its *relative weight or specific gravity* may be determined.

192. The *specific gravity* of a body is its weight as compared with the weight of an equal bulk or volume of some other body assumed as a standard.

193. Pure distilled water at the temperature of 60° Fahr. is taken as the standard with which to compare all solids and liquids, and pure dry atmospheric air at a temperature of 32° Fahr., and a barometric pressure of 30 inches is taken as the standard with which all gases are compared.

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194. To find the specific gravity of a solid heavier than water :—

RULE.

Divide the weight of the body in air by its loss of weight in water, the result will be its specific gravity.

Example 251.—A piece of lead weighs 225 grains in air, and only 205 grains in water; required its specific gravity.

SOLUTION.

Loss of weight = 225 — 205 = 20 grains.
Hence specific gravity = 225 ÷ 20 = 11·250. *Ans.*

Example 252.—A piece of sulphur weighs 97 grains in air and but 50·5 grains in water; what is its specific gravity?

SOLUTION.

Loss of weight in water = 97 — 50·5 = 46·5 grains.
Then specific gravity = 97 ÷ 46·5 = ·008. *Ans.*

EXERCISE.

253. A piece of silver weighs 200 grains in air and only 180 grains in water; required its specific gravity. *Ans.* 10·000.

254. A piece of platinum weighs 154½ oz. in air and only 147½ oz. in water; required its specific gravity. *Ans.* 22·071.

255. A piece of glass weighs 193 oz. in air and but 130 oz. in water; required its specific gravity. *Ans.* 3·063.

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195. To find the specific gravity of a solid not sufficiently heavy to sink in water.

RULE.

To the body whose specific gravity is sought attach some other body sufficiently heavy to sink it, and of which the weight in air and loss of weight in water are known.

Then weigh the united mass in water and in air, from its loss of weight deduct the loss of weight of the heavier body in water, and divide the absolute weight of the lighter body by the remainder, the quotient will be the specific gravity of the lighter body.

Example 256.—A piece of wood which weighs 55 oz. in air has attached to it a piece of lead which weighs 45 oz. in air and 41 in water, the united mass weighs 30 oz. in water; required the specific gravity of the piece of wood.

SOLUTION.

Wt. of united mass in air = 55+45 = 100 oz.
" " water = 30 "

Loss of wt. of united mass in water = 70 "
Loss of wt. of lead in water = 4 "

Remainder = 66 " = loss of weight of the wood.
Then 55 ÷ 66 = 833 = specific gravity of the wood.

EXAMPLE 257.—A piece of wood which weighs 70 oz. in air has attached to it a piece of copper which weighs 36 oz. in air and 31·5 oz. in water, the united mass weighs only 11·7 oz. in water; what is the specific gravity of the wood?

SOLUTION.

Wt. of united mass in air = 70+36 = 106 oz.
" " water = 11.7 "

Loss of wt. of united mass in water = 94·3 "
Loss of wt. of copper " = 4·5 "

Loss of wt. of wood " = 89·8 " = loss of weight of the wood.
Then specific gravity of wood = 70 ÷ 89·8 = 779. *Ans.*

EXERCISE.

258. A piece of pine wood which weighs 15 lbs. in air has attached to it a piece of copper which weighs 18 lbs. in air and 16 lbs. in water; the weight of the united mass in water is 6 lbs.; required the specific gravity of the pine? *Ans.* ·600.

259. A piece of cork which weighs 20 oz. in air has attached to it an iron sinker which weighs 18 oz, in air and 15 73 oz. in water, the united mass weighs 1 oz. in water; required the specific gravity of the cork? *Ans.* ·575.

260. A piece of wood which weighs 33 oz. in air has attached to it a metal sinker which weighs 21 oz. in air and 18·19 oz. in water, the united mass weighs 2·5 oz. in water; what is the specific gravity of the wood? *Ans.* ·677.

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196. The specific gravities of liquids may be determined in three different ways.

FIRST METHOD.—*A small glass flask, which holds precisely* 1000 *grains of pure distilled water at the temperature of* 60° *Fahr., is filled with the liquid in question and accurately weighed, the result indicates the specific gravity of the liquid.*

SECOND METHOD.—*A piece of substance of known specific gravity is weighed both in and out of the liquid in question. The difference of weight is multiplied by the specific gravity of the solid,* *and the product divided by the absolute weight of the solid, and the result is the specific gravity of the liquid.*

That is $s = \frac{w - w'}{w} \times s$;

where w = *absolute weight of solid.*
w' = *weight in the liquid.*
Therefore $w - w'$ = *loss of weight.*
s = *specific gravity of the liquid.*
s' = *specific gravity of the solid.*

THIRD METHOD.—*This specific gravity of liquids is most commonly found in practice by means of an instrument called the* HYDROMETER, *which consists of a graduated scale rising from a glass or silver bulb, beneath which is a small appendage loaded with shot or some other heavy substance. It acts upon the principle that the greater the density of a liquid the greater will be its specific gravity. The depth to which the instrument sinks in different liquids is shown by the graduated scale, which thus indicates their specific gravities. For liquids specifically lighter than water, the scale is graduated from the bottom upwards; for those heavier, from the top downwards.*

Fig. 19.

EXAMPLE 261.—The Thousand-grain Bottle filled with sulphuric acid weighs 1841 grains.* What is the specific gravity of the sulphuric acid?

SOLUTION.

$1841 \div 1000 = 1{\cdot}841$. *Ans.*

EXAMPLE 262.—The Thousand-grain Bottle filled with alcohol weighs 792 grains, required the specific gravity of alcohol.

SOLUTION.

$792 \div 1000 = {\cdot}792$. *Ans.*

EXAMPLE 263.—A piece of zinc (specific gravity 7·190) weighs 27·4 oz. in a certain liquid and 32·7 oz. out of it, required the specific gravity of the liquid.

SOLUTION.

Here $w = 32{\cdot}7$, $w' = 27{\cdot}4$, $s' = 7{\cdot}190$.

Then $s = \frac{w-w'}{w} \times s' = \frac{32{\cdot}7-27{\cdot}4}{32{\cdot}7} \times 7{\cdot}190 = \frac{5{\cdot}3 \times 7{\cdot}190}{32{\cdot}7} = 1{\cdot}165$. *Ans.*

EXAMPLE 264.—A piece of silver (specific gravity 10·500) weighs 47·8 grains in a liquid and 58·2 grains out of it—what is the specific gravity of the liquid?

* That is not including the weight of the bottle itself.

SOLUTION.

Here $w = 58{\cdot}2$, $w' = 47{\cdot}8$ and $s' = 10{\cdot}5$.

Then $s = \frac{w-w'}{w} \times s' = \frac{58{\cdot}2-47{\cdot}8}{58{\cdot}2} \times 10{\cdot}5 = \frac{10{\cdot}4\times10{\cdot}5}{58{\cdot}2} = 1{\cdot}876$. *Ans*

EXERCISE.

265. A piece of copper (specific gravity 8·850) weighs 446·3 grains in liquid, and 490 grains out of it, required the specific gravity of the liquid. *Ans.* ·789.

266. The Thousand-grain Bottle filled with olive oil weighs 915 grains—what is the specific gravity of olive oil? *Ans.* ·915.

267. The Thousand-grain Bottle filled with mercury weighs 13596 grains—what is the specific gravity of mercury? *Ans.* 13·596.

268. A piece of cast-iron (specific gravity 7·425) weighs 34·61 oz. in a liquid, and 40 oz. out of it,—what is the specific gravity of the liquid? *Ans.* 1.000 nearly.

269. A piece of gold (specific gravity 19·360) weighs 139·85 grains in a liquid, and 159·7 grains in the air, required the specific gravity of the liquid? *Ans.* 2·406.

270. A piece of marble (specific gravity 2·850) weighs 30 lbs. in a certain liquid, and 35·9 lbs. in the air, required the specific gravity of the liquid? *Ans.* ·468.

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197. The specific gravity of gases is found by exhausting a flask of atmospheric air and filling it with the gas in question previously well dried. This is accurately weighed and its weight compared with the weight of the same volume of dry atmospheric air at the temperature of 60° Fahr. and under a barometric pressure of 30 inches.

198. The following table gives the specific gravities of the most common substances:

TABLE OF SPECIFIC GRAVITIES.

GASES.		Copper,..............	8·850
Atmospheric air,......	1·000	Brass,..............	8·300
Hydrogen,...........	·069	Iron,................	7·788
Oxygen..............	1·106	Tin,................	7·293
Nitrogen,...........	·972	Zinc,...............	7·190
Ammoniacal gas,......	·596	Diamond,............	3·530
Carbonic acid gas,....	1·529	Flint glass,..........	3·330
Sulphurous acid gas,...	2·234	Sulphur,.............	2·086
Chlorine,............	2·470	Slate,..............	2·840
LIQUIDS.		Brick,..............	2·000
Distilled water,.......	1·000	Common stone........	2·460
Mercury..............	13·596	Marble,.............	2·850
Sulphuric acid........	1·841	Ivory,...............	1.825
Nitric acid,...........	1·220	Phosphorus,..........	1·770
Milk,................	1·030	Lignum vitæ,.........	1·350
Sea water,...........	1·026	Boxwood.............	1·320
Wine,...............	·993	Potassium,...........	·8 75
Olive oil.............	·915	Sodium,.............	·972
Spirits of turpentine...	·869	Pumice stone,........	·914
Pure alcohol,.........	·792	Dry pine,............	·657
Ether,...............	·715	Dry poplar,..........	·383
Prussic acid,..........	·696	Ice,.................	·865
SOLIDS.		Living man,..........	·891
Platinum............	22·050	Cork,...............	·240
Gold,...............	19·360	Graphite,............	2·500
Silver,..............	10·500	Bituminous coal,......	1·250
Lead,...............	11·250	Anthracite coal,......	1·800

199. A cubic foot of pure distilled water at the temperature of 60° Fahr. weighs exactly 1000 ounces. Hence if the specific gravity of any substance be known, the weight of a cubic foot, &c., may be easily found.

For example.—The specific gravity of mercury is 13·596 water, being 1·000 and a cubic foot of water weighing 1000 ounces, it follows that a cubic foot of mercury weighs 13596 ounces.

200. To find the solid contents of a body from its weight:—

RULE

Contents in feet $= \frac{w}{w'}$; *where* w = *whole weight, and* w' = *weight of a cubic foot as ascertained from its specific gravity.*

EXAMPLE 271.—How many cubic feet are there in 2240 lbs. of dry oak (specific gravity ·925.)?

SOLUTION.

Here $\frac{w}{w'} = \frac{2240 \text{ lbs.}}{925 \text{ oz.}} = \frac{35840}{925} = 38\frac{138}{185}$ cubic feet.

EXAMPLE 272.—How many cubic feet are there in a mass of iron which weighs 17829 lbs.?

SOLUTION.

Specific gravity of iron = 7·788. Therefore 1 cubic foot weighs 7788 oz.
Then cubic feet in mass = 17829 lbs. ÷ 7788 oz. = 36·628. *Ans.*

201. To find the weight of a body from its solid contents :—

$w =$ *contents in ft.* $\times\ w'$.

RULE.

Where w and w′ are same as in last rule.

EXAMPLE 273.—What is the weight of a block of dry oak 10 ft. long, 3 ft. thick, $2\frac{1}{2}$ ft. wide?
Here $10 \times 3 \times 2\frac{1}{2} = 75$ cubic feet.
Then $w = w' \times 75 = 925 \text{ oz.} \times 75 = 69375 \text{ oz.} = 4335\frac{15}{16}$ lbs. *Ans.*

EXAMPLE 274.—What is the weight of a block of marble 8 ft. long. 2 ft. wide, $1\frac{1}{2}$ ft. thick.

SOLUTION.

Cubic feet of marble = $8 \times 2 \times 1\frac{1}{2} = 24$.
Spec. grav. of marble = 2·850. Therefore one cubic foot weighs 2850 oz.
Then weight of block = $2850 \times 24 = 68400$ oz. = 4275 lbs. *Ans.*

EXERCISE.

275. What is the weight of a mass of copper which contains 29 cubic feet? *Ans.* 16040 lbs. 10 oz.

276. How many cubic feet are there in a mass of lead which weighs seven million pounds? *Ans.* 9955·55 cub. ft.

277. How many cubic feet of sulphuric acid are there in 78124732 lbs.? *Ans.* 678976·48 cub. ft.

278. What is the weight of the mercury contained in a rectangular cistern 6 feet long, 4 feet wide and 10 feet deep the mercury filling it? *Ans.* 203940 lbs.

279. If a block of zinc be 11 feet long by 3 feet wide and 2 feet thick, how much does it weigh? *Ans.* $29658\frac{3}{4}$ lbs.

280. What is the weight of a squared log of dry pine 44 feet long and 18 inches square? *Ans.* 4065 lbs. 3 oz.

202. In order that a floating body may be in equilibrium it is requisite that :—

1st. The weight of the water displaced shall be equal to the weight of the floating body; and,

2nd. The resultant of all the upward pressures of the liquid shall act in the line of direction of the centre of gravity of the body.

203. The centre of buoyancy of a floating body is the point upon which the resultant of all the upward pressures of the liquid acts.

NOTE.—The centre of buoyancy coincides not with the centre of gravity of the floating body, but with the centre of gravity of the fluid displaced. While the body floats, the centre of buoyancy is always below the centre of gravity, but the two coincide when the body sinks. In a ship, however, or other hollow body, containing much heavy ballast in the hold, the centre of gravity is below the centre of buoyancy.

204. A floating body is in equilibrium when the centre of gravity and the centre of buoyancy are in the same vertical line, and the equilibrium is :—

Stable when the centre of gravity is below the centre of buoyancy.

Neutral when the centre of gravity coincides with the centre of buoyancy.

Unstable when the centre of gravity is above the centre of buoyancy.

CHAPTER VI.

PNEUMATICS.

205. Pneumatics treats of the mechanical properties of *permanently elastic fluids*, of which atmospheric air may be taken as the type.

206. The atmosphere (Greek *atmoi* "gases") or sphere of gases is the name applied to the gaseous envelope which surrounds the earth.

207. It is supposed, from certain astronomical considerations that the atmosphere extends to the height of only about 45 miles above the surface of the earth.

NOTE.—The height of the atmosphere is only about $\frac{1}{90}$ of the radius of the earth, so that upon an artificial globe 24 inches in diameter the atmosphere would be represented by a covering $\frac{1}{8}$ of an inch in thickness.

208. Atmospheric air is a mechanical mixture chiefly of two gases, oxygen and nitrogen, in the proportion of

1 gallon of the former to 4 gallons of the latter. Its exact composition, omitting the aqueous vapor, is as follows:—

COMPOSITION BY VOLUME.

Nitrogen,	79·12	per cent.
Oxygen,	20·80	"
Carbonic acid,	·04	"
Carburetted Hydrogen,	·04	"
Ammonia,	*Trace.*	

NOTE.—*Oxygen* is the sustaining principle of animal life and of ordinary combustion. When an animal is placed in a vessel of pure oxygen its heart beats with increased energy and rapidity and it very soon dies from excess of vital action. Many substances, also, that are not all combustible under ordinary circumstances burn when placed in pure oxygen with extraordinary brilliancy and vigor.

Nitrogen, on the other hand, supports neither respiration nor combustion. In its chemical nature it is distinguished chiefly by its negative properties. In the atmosphere it serves the important purpose of diluting the oxygen and thus fitting it for the function it is designed to perform in the animal economy.

Carbonic acid is a highly poisonous gas, formed by the union of oxygen and carbon (charcoal). It is produced in large quantities during the processes of animal respiration, common combustion, fermentation, volcanic action and the decay of animal and vegetable substances. Although when inhaled, it rapidly destroys animal life it constitutes the chief source of food to the plant. Animals take into the lungs air loaded with oxygen and throw it off so charged with carbonic acid as to be incapable of again serving for the purposes of respiration. The green parts of plants, on the contrary, absorb air, decompose the carbonic acid it contains, retain the carbon and give off air containing no carbonic acid but a large amout of oxygen. This is a most beautiful illustration of the mutual dependence of the different orders of created beings upon one another. Were it not for plants, the air would rapidly become so vitiated as to cause the total extinction of animal life; were it not for animals, plants would not thrive for want of the food now supplied in the form of carbonic acid by the living animal. A sit is, the one order of beings prepares the air for the sustenance and support of the other, and so admirably is the matter adjusted that the composition of the air is, within very narrow limits, invariably the same.

The amount of carbonic acid varies from 3·7 as a minimum to 6·2 as a maximum in 10000 volumes.

Carburetted Hydrogen is produced during the decay of animal and vegetable substances. It is one of the chief ingredients of common illuminating gas, and is poisonous to animals when present in the air in large quantities.

209. One of the most remarkable characteristics of gases, is the property they possess of diffusing themselves among one another. Thus if a light gas and a heavy one are once mixed they exhibit no tendency to separate again, and no matter how long they may be allowed to stand at rest, they are found upon examination intimately mingled with each other. Moreover if two vessels be placed one upon the other, the upper being filled with any light gas (hydrogen) and the lower with any heavy gas (carbonic acid), and if the two gases be allowed to communicate with one another by a narrow tube, or a porous membrane, a remarkable interchange rapidly takes place, i.e., in direct opposition to the attraction of gravity the heavy gas ascends and the light gas descends until they become perfectly mixed in both vessels.

NOTE.—The property of *gaseous diffusion* has a very intimate bearing upon the composition of the air. If either of the constituents of the air were to separate from the mass, the extinction of life would soon follow. Besides were it not for the existence of this property, various vapors would accumulate in certain localities, as large cities, manufacturing districts, volcanic regions, &c., in such quantities as to render them totally uninhabitable.

210. In addition to the gases already mentioned, atmospheric air always contains more or less water in the form of invisible vapor. This is derived partly from combustion, respiration and decay, but chiefly from spontaneous evaporation from the surface of the earth. The amount of invisible vapor thus held in solution depends upon the temperature of the air being as high as $\frac{1}{30}$ of the weight of the air in very hot weather, and as low as $\frac{1}{510}$ in cold.

211. The blue color of the sky is due to light that has suffered polarization, and which is, therefore, reflected light, like the white light of the clouds. The air appears to absorb to a certain extent the red rays and yellow rays of solar light and to reflect the blue rays. In the higher regions the blue becomes deeper in color and is mixed with black. The golden tints of sunset depend upon the large amount of aqueous vapor held in solution by the air.

212. Air, like all other material bodies, possesses the properties of impenetrability, extension, inertia, porosity, compressibility, elasticity, &c. (See Arts. 11-18.)

NOTE 1.—The *impenetrability* of atmospheric air is illustrated by various experiments, among which are the following:

I. If an inverted tumbler be immersed in water the liquid does not rise in the interior of the tumbler, because the latter is full of air and the water cannot enter until the air has been displaced.

II. If the two boards of a bellows be drawn asunder and while in that position the nozzle of the bellows be closed, the boards cannot be pressed together because the bellows is full of air.

III. If an india-rubber bag or a bladder be inflated with air, and pressure applied, it is found that there is a *material something* within which keeps the sides asunder,—that material something is atmospheric air.

NOTE 2.—The *Inertia* of atmospheric air is shown:—
I. By the force of wind, which is nothing more than air in motion.
II. By attempting to run on a calm day, carrying an open umbrella.
III. By the apparent current of wind experienced on a perfectly calm day by a person standing on the deck of a steamboat, or the platform of a railway car when in rapid motion, which current is caused by the body *displacing* the air.

IV. By causing a feather and a ball of lead to fall in a vacuum, when it is observed that they fall with the same velocity. In the atmosphere, however, the ball falls faster than the feather because it contains a greater amount of matter with the same extent of surface as the feather, and hence, meets with less resistance from the inertia of the air.

213. Air, in common with all other forms of matter, is acted on by the attraction of gravity, and consequently possesses *weight*.

NOTE 1.—To prove this is the *fundamental fact* in the science of pneumatics, we take a glass globe capable of containing 100 cubic inches, and after weighing it accurately, withdraw from it, by means of an air pump, all the air it contains. When we weigh it again we find that its weight is about 31 grains less than when filled with air.

100	cubic inches of	Atmospheric air	weigh	 31	grains.
100	"	Oxygen	"	 34	"
100	"	Nitrogen	"	 30	"
100	"	Carbonic acid	"	.... 47¼	"
100	"	Hydrogen	"	 2	"

NOTE 2.—Although a small quantity of air when examined appears to be almost imponderable, the aggregate weight of the entire atmosphere is enormous, being equal to:
I. Five thousand millions of millions of tons; or
II. A globe of lead 66 miles in diameter; or
III. An ocean of water covering the whole surface of the earth to the depth of 32 feet; or
IV. A stratum of mercury covering the entire surface of the globe to the depth of 30 inches.

214. Since the air is ponderable and also compressible, and since the lower stratum has to sustain the pressure of the superincumbent portion, it necessarily follows that the air is denser near the surface of the earth than in the higher regions of the atmosphere.

215. The density of the air decreases in geometrical progression, while the elevation increases in arithmetical progression. That is at the height of 2·7 miles, the atmospheric pressure is reduced to one-half, at twice that height to one-fourth, at three times that height to one-eighth, &c.

NOTE.—The following table exhibits the density, elasticity and pressure of the air at the different elevations given. Halley fixed the height at which the pressure is decreased to one-half at 3½ miles, but a more careful collection, by Biot and Arago of the observations made on the Andes and in balloons respecting the upward decrease of pressure and temperature, has led to the adoption of 2·7 miles as the point at which we may say that one-half of the atmosphere is beneath us.

HEIGHT IN MILES.	DENSITY.	HEIGHT, IN IN., OF COLUMN OF MERCURY	PRESSURE IN LBS. TO THE SQ. INCH.
2·7	$\frac{1}{2}$	15	7·5
5·4	$\frac{1}{4}$	7·5	3·75
8·1	$\frac{1}{8}$	3·75	1·875
10·8	$\frac{1}{16}$	1·875	·937
13·5	$\frac{1}{32}$	·937	·468
16·2	$\frac{1}{64}$	·468	·234
18·9	$\frac{1}{128}$	·234	·117
21·6	$\frac{1}{256}$	·117	·058
24·3	$\frac{1}{512}$	·058	·029
27·0	$\frac{1}{1024}$	·029	·014
29·7	$\frac{1}{2048}$	·014	·007

216. The pressure of the air is a necessary consequence of its weight, and is equal, at the level of the sea, to about 15 lbs. to the square inch.

NOTE.—By saying that the pressure of the atmosphere is equal to 15 lbs. to the sq. inch, we mean that it is capable of balancing a column of mercury 30 inches in height; and a column of mercury 30 inches in height and having a sectional area of 1 sq. inch, weighs 15 lbs. Or in other words, that a column of air having a sectional area of 1 sq. inch, and extending from the level of the sea to the top of the atmosphere, weighs 15 lbs.

217. Air at 60° F. is 810 times as light as water, and 10466 times as light as mercury. It follows that the pressure of the atmosphere is equal to that of a column of air of the same density as that at the surface of the earth 810 times 32 feet or 10466 times 30 inches in height. That is, if the air were throughout of the same density that it is at the level of the sea, it would extend to the height of about 5 miles.

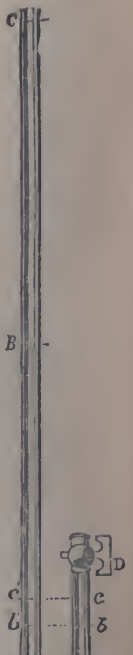
218. The particles of elastic gases, unlike those of solids or liquids, possess no cohesive attraction, but on the contrary a powerful repulsion, by means of which they tend to separate from one another as far as possible.

219. Permanently elastic fluids, such as atmospheric air, and certain gases, are chiefly distinguished from non-elastic fluids, such as water, by the possession of almost perfect elasticity and compressibility.

NOTE.—Air and certain gases as Oxygen, Hydrogen, Nitrogen, &c., are called *permanently* elastic to distinguish them from a number of others as Carbonic Acid, Nitrous Oxide, &c., which under great pressure and intense cold pass first into the liquid and finally into the solid state.

220. If a liquid be placed in a cylinder under the piston, it will remain at the same level, no matter to what height the piston may be raised above it, but if a portion of air or any other elastic gas be thus placed in the cylinder, and the piston be air-tight, the confined air will expand upon raising the piston and will always fill the space beneath it, however great this may become. This expansibility or tendency to enlarge its volume so as to entirely fill the space in which it is enclosed is termed *elasticity*.

Fig. 20.



NOTE.—It is obvious that the elasticity of air is due to the repulsive power possessed by the particles.

221. The law determining the density and elasticity of gases under different pressures was investigated by Boyle in 1660, and afterwards by Mariotte.

NOTE.—To illustrate this law we take a bent glass tube Fig. 20, having one limb AC much longer than the other. The longer limb is open and the shorter furnished with a stop-cock.

Both ends being open a quantity of mercury is poured into the tube and of course rises to the same level in both legs—the surface of the mercury at *A a*, sustaining the weight of a column of air extending to the top of the atmosphere. We now close the stop-cock and thus shut off the pressure of the atmosphere above that point, so that the surface *a*, cannot be affected by the weight of the atmosphere—i.e., cannot be influenced by atmospheric pressure. We find, however, that the mercury in both limbs remains at the same level, from which we infer that the elastic force of the air confined above *a* is equal to the weight of the whole column on *a* before the stop-cock was closed.

Hence the elasticity of the air is equal to its weight which is equal to a column of mercury 30 inches high.

If now we pour mercury into the tube until the air confined above *a* is compressed into half its former volume, i.e., until the mercury rises to *b* in the shorter tube, we shall find that the column of mercury *b B* is exactly 30 inches in length, or in other words, we have doubled the pressure on the air confined in the shorter tube and have decreased its volume to one-half its former dimensions, and at the same time doubled its elastic force, since it now reacts against the surface of the mercury with a force equal to 30 lbs. to the square inch.

If we increased the height of the Mercury in the longer leg to 60 inches above its height in the shorter leg, we shall compress the air into one-third its original volume and at the same time treble its elasticity, and so on. Hence the law of Mariotte.

222. Mariotte's law may be thus enunciated:

I. The density and elasticity of a gas vary directly as the pressure to which it is subjected.

II. The volume which a gas occupies under different pressures varies inversely as the force of compression.

NOTE—Recent researches tend to prove that Mariotte's law is true only within certain limits, and that all gases vary from the law when subjected to very great pressures, their density increasing in a greater ratio than their elasticity. With atmospheric air the law holds good to a far greater extent than with any other gas, the correspondence being found to be rigidly exact when the air is expanded to 300 volumes, and also when it is compressed into $\frac{1}{25}$ of its primary volume.

Mariotte's law would require the air to be indefinitely expansible, while we know that there is, beyond all doubt, an upward limit to the atmosphere. Dr. Wollaston imagines that when the particles of air are driven a certain distance apart by their mutual repulsive power, the weight of the individual particles comes at last to balance this repulsive force, and thus prevent their further divergence. If this be the case, as is probable from various considerations, there is a limit to the rarefaction of a gas, arriving at which the gas ceases to expand further and comes to have a true upper surface like a liquid. As has been already remarked, this exact limit and upper surface of the atmosphere is supposed to be at an elevation certainly not greater than 45 miles—Biot fixes it at 30 miles; Bunsen and others place it at 200 miles.

223. The air-pump, as its name implies, is an instrument used for pumping out or exhausting the air from any closed vessel.

224. The bell-shaped glass vessel usually attached to the air-pump is called a *Receiver*, and when the air is exhausted as far as practicable from this, a *vacuum* is said to have been produced.

NOTE.—The air-pump was invented by Otto Guericke, a celebrated burgomaster of Magdeburg, in the year 1560. At the close of the Imperial Diet in 1564, he exhibited his first public experiments with it before the emperor and assembled princes and nobles of Germany. On this occasion he exhausted the air from two 12-inch hemispheres fitted together by ground edges, and greatly astonished his noble audience by showing that the combined strength of 12 horses was insufficient to pull them asunder.

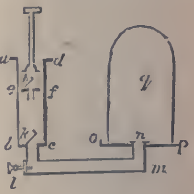
The exhausting syringe of Otto Guericke was so imperfect in its action that while using it he was compelled to keep it immersed in water to prevent the inward leakage of the air. Since his time, however, the attention of many eminent men has been directed to the subject, and the form and construction of the air-pump have been greatly improved.

225. The *exhausting syringe*, which is the essential part of an air-pump, consists of a brass cylinder *abcd*, supplied

with an air-tight piston *ef*, and an arrangement of valves *h'k*, by means of which the air is permitted to pass out from the receiver *q* and through the piston *ef*, but not in the contrary direction.

NOTE.—When the piston *ef* is raised the valve *h* closes, and as the piston in its ascent produces a partial vacuum beneath it, the air contained in the receiver *q* opens the valve *k* by its expansive power and thus refills the cylinder *abcd*. Now when the piston is forced down again, the air contained in the cylinder tends to rush back into the receiver, but in doing so closes the valve *k*, and has therefore no other mode of escape than through *h*, thus passing above the piston to be lifted out at the next stroke. In this manner the air continues to be exhausted until what remains in the receiver has not sufficient expansive power to open the valve *k*, when the exhaustion is said to be complete.

Fig. 21.



226. The principle upon which the air-pump acts is the elasticity or expansibility of the air, and since in order to enable the pump to act, the air contained in the receiver must possess sufficient elastic force to raise the valve, it follows that a perfect vacuum cannot be secured by the air-pump. Thus, pumps of common construction will not withdraw more than $\frac{99}{100}$ of the contained air, but the improved form is said to exhaust $\frac{9999}{10000}$.

NOTE.—If we suppose the cylinder of the exhausting syringe to have the same effective capacity as the receiver, and that the piston passes at each stroke the whole length of the cylinder, it is evident that in raising the piston to the top of the cylinder and then depressing it again to the bottom, one-half of the air will have passed from the receiver; the remaining half completely filling it, but having only half as much density and elasticity as before. The second stroke of the piston will reduce the quantity, density, and elasticity, to one-fourth, the third to one-eighth, and so on, as exhibited by the following table:

Stroke.	*Goes out.*		*Left in Vessel.*	*Elastic force of the remainder*					
1st,	$\frac{1}{2}$ of 1	=	$\frac{1}{2}$	15	in. of mercury,	or	7·35 lbs.	per	sq. in.
2nd,	$\frac{1}{2}$ of $\frac{1}{2}$	=	$\frac{1}{4}$	7½	"	"	" 3·675	"	"
3rd,	$\frac{1}{2}$ of $\frac{1}{4}$	=	$\frac{1}{8}$	3¾	"	"	" 1·837	"	"
4th,	$\frac{1}{2}$ of $\frac{1}{8}$	=	$\frac{1}{16}$	1·875	"	"	" ·918	"	"
5th,	$\frac{1}{2}$ of $\frac{1}{16}$	=	$\frac{1}{32}$	0·937	"	"	" ·459	"	"
6th,	$\frac{1}{2}$ of $\frac{1}{32}$	=	$\frac{1}{64}$	0·468	"	"	" ·229	"	"
7th,	$\frac{1}{2}$ of $\frac{1}{64}$	=	$\frac{1}{128}$	0·234	"	"	" ·114	"	"
8th,	$\frac{1}{2}$ of $\frac{1}{128}$	=	$\frac{1}{256}$	0·117	"	"	" ·057	"	"
9th,	$\frac{1}{2}$ of $\frac{1}{256}$	=	$\frac{1}{512}$	0·058	"	"	" ·028	"	"

227. The *condensing syringe*, which is used for forcing air into a receiver or condensing chamber, differs from an

exhausting syringe only in the fact that its valves open inward towards the chamber instead of outward.

228. The air pump is chiefly employed to illustrate the pressure and elasticity of the air.

NOTE 1.—The *pressure* of the atmosphere may be shewn by innumerable experiments among which are the following:

I. When the air is exhausted from the receiver of an air-pump the receiver is firmly fastened to the plate, and cannot be removed until the air is re-admitted.

II. The hand placed on the open end of the receiver is pressed inward with a force sufficiently great to cause pain.

III. Thin square glass-tubes are crushed when the air is exhausted from them.

IV. In the surgical operation of *cupping*, the air is removed from a small cup which is then placed over an opened vein; the pressure of the air on the surrounding parts causes the blood to flow rapidly into the cup.

V. When a cask of beer is tapped, the beer does not run until a small hole called the *vent-hole* has been made in the upper part of the cask. Through this the atmospheric air enters and pressing on the surface of the beer with a force of 15 lbs. to the square inch, forces it through the tap.

VI. The useful small glass instruments called *pipettes* act upon the principle of atmospheric pressure.

VII. A hole is usually made in the lid of a tea-pot so as to bring into play the pressure of the atmosphere and thus cause the beverage to flow more rapidly.

VIII. Flies walk on glass or on the ceiling by producing a vacuum under each foot which is thus pressed against the surface with a force sufficient to sustain the weight of the insect. The gecko, a South American lizard, has a similar apparatus attached to each foot. And within the past few years a man has succeeded in walking across a ceiling with his head downwards, by alternately withdrawing and admitting the air between his feet and the ceiling.

IX. Pneumatic chemistry, i. e., the mode of collecting gases over water depends upon the principle of atmospheric pressure.

X. If a tumbler or other glass vessel be filled with water and covered with a piece of paper, and the hand be then placed firmly on the paper and the whole suddenly and carefully inverted, the water does not flow out of the vessel upon removing the hand—being held by the upward pressure of the atmosphere.

XI. Suction is the effect of atmospheric pressure, as illustrated by *drawing* liquids into the mouth, also by the *leather sucker* used by boys.

XII. The pressure of the air is shown by the fact that it supports or balances a column of mercury 30 inches or a column of water 32 feet in height.

XIII. The pressure of the atmosphere retards ebullition or boiling. Thus if some boiling water be partially cooled and then placed under the receiver of an air pump and the air exhausted, the water recommences to boil, owing to the decreased pressure. Or if a flask containing boiling water be corked and the water be allowed to cool partially, upon plunging the flask in a large vessel of cold water, the water in the flask again begins to boil; the reason is, the cold water condenses the vapor in the upper part of the flask and thus produces a partial vacuum.

NOTE 2—The elasticity of the air may be shown by various experiments among which are the following:

I. The exhaustion of the receiver of the air-pump is a proof of the elasticity of the air.

II. The elasticity of the air is shown by placing a thin square bottle with its mouth closed, under the receiver, and exhausting the surrounding air, the bottle is broken by the elastic force of the contained air.

III. When some withered fruit, as apples, figs, or raisins, with unbroken skins are placed under the receiver, and the surrounding air exhausted, they become plump from the elasticity of the included air.
IV. The elasticity of air is shewn by the operation of the air-gun.
V. The elasticity of the air is taken advantage of in applying air as a stuffing material for cushions, pillows, and beds.

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229. The *barometer* (Greek *baros* "weight" and *metreo* "I measure") is an instrument designed to measure the variations in the amount of atmospheric pressure.

NOTE.—The barometer was invented about the middle of the seventeenth century by Torricelli, a pupil of the celebrated Galileo.

230. The essential parts of a barometer are:

1st. A well formed glass tube 33 or 34 inches long, closed at one end and having a bore equal throughout, of two or three lines in diameter. The tube contains pure mercury only, and is so arranged that the mercury is supported in the tube by the pressure of the atmosphere; and

2nd. An attached graduated scale and various appliances for protecting the tube and ascertaining the exact height of the column of mercury.

NOTE.—The vacant space between the top of the column of mercury and the top of the tube is called the *Torricellian vacuum*, in honor of the inventor of the barometer, and in a good instrument is the most perfect vacuum that can be produced by mechanical means.

231. The excellency of a barometer depends principally upon the purity of the mercury in the tube, and the perfectness of the Torricellian vacuum.

The value of the instrument may be tested:—
1st. By the brightness of the column of mercury, and the absence of any speck, flaw, or dullness on its surface.
2nd. By the *barometric light*; i. e., flashes of electric light produced in the dark in the Torricellian vacuum by the friction of the mercury against the glass.
3rd. By the clearness of the ring or clicking sound produced by making the mercury strike the top of the tube, and which is greatly modified when any particles of air are present above the column.

232. The cause of all the oscillations in the barometer is to be found in the unequal and constantly varying distribution of heat over the earth's surface. If the air is much heated at any spot it expands, rises above the mass of air, and rests upon the colder portions surrounding it. The ascended air consequently flows off laterally from above, the pressure of the air is decreased in the warmer place and the barometer falls. In the colder surrounding places, however, the barometer rises, because the air that ascended in the warmer regions is diffused over and presses upon the atmosphere of these cooler parts.

NOTE.—It is found that the fluctuations in the height of the barometer vary greatly in extent in different latitudes—being so small in tropical regions as almost to escape notice, and comparatively so fitful and extreme in the temperate and frigid zones as to defy all attempts at reducing them to any system. In our climate the column varies in height from a little over 30 inches as a maximum, to a little over 27 inches as a minimum. Within the torrid zone the column of mercury scarcely ever exhibits any disturbance greater than what would occur in Canada before a slight thunder storm—but such a disturbance is there the sure and rapid precursor of one of those mighty atmospheric convulsions which sometimes desolate vast regions and which are frequently as disastrous in their effects as the most violent earthquakes.

233. Besides the irregular fluctuations depending upon the weather, the barometer is subject to regular *semi-diurnal oscillations* depending upon atmospheric tides, caused by the *heat* of the sun—the two *maxima* of pressure always occurring at about 9 a.m. and 9 p.m. and the two *minima* at about 3 a.m. and 3 p.m.

NOTE.—The semi-diurnal oscillation is greatest at the equator, where it averages one-tenth ot an inch—diminishing to *six hundredths* of an inch in lat. 30°, beyond which it still decreases, and in our climate becomes completely masked by the irregular fluctuations peculiar to the temperate and frigid zones.

234. USE OF THE BAROMETER AS A WEATHER-GLASS.

I. *The state of the weather to be expected depends not so much upon the absolute height of the column of mercury as upon the* RAPIDITY AND EXTENT OF ITS MOTION *whether rising or falling.*

NOTE.—If the mercury have a convex surface, the column is rising; if the surface is concave, the column is falling; when the surface is flat, the column is usually changing from one of these states to the other.

II. *A fall in the barometer generally indicates approaching rain, high winds, or a thunder storm.*

III. *A rise in the mercury commonly indicates the approach of fine weather; sometimes, however, it indicates the approach of a snow storm.*

IV. *A rapid rise or fall in the mercury indicates a sudden change of weather.*

V. *A steady rise in the column, continued for two or three days, is generally followed by a long continuance of fine settled weather.*

VI. *A steady fall in the column, continued for two or three days, is commonly followed by a long continuance of rainy weather.*

VII. *A fluctuating state in the height of the mercury coincides with unsettled weather.*

NOTE.—The barometer is far more valuable as a means of ascertaining approaching changes in the state of the wind than in foretelling the approach of wet or dry weather.

235. To ascertain the height of mountains, &c., by the barometer.

HALLEY'S RULE.

I. *Find the logarithm corresponding to the number which expresses the height in inches of the column of mercury in the barometer at the level of the sea.*

II. *Find also the logarithm corresponding to the number which expresses in inches the height of the column in the barometer at the top of the mountain or other given elevation.*

III. *Subtract the latter of these logarithms from the former, multiply the remainder by the constant number, 62170, and the result will be the elevation in English feet.*

NOTE.—The number 62170 in this rule and 63946 in the following, were selected by Halley for certain mathematical reasons into which it is unnecessary to enter.

EXAMPLE 281.—On the top of a certain mountain the barometer stands at the height of 21·793 inches, while on the surface of the earth it stands at 29·780 inches; required the height of the mountain.

SOLUTION.

Logarithm of 29·780 = 1·473925 and logarithm of 21·793 = 1·328317.

Then from 1·473925
Subtract 1·328317

Remainder = ·145608 × 62170 = 9052 feet, *Ans*

RULE WITH CORRECTION FOR TEMPERATURE.

I. *Obtain, as before, the difference between the logarithms of the numbers expressing the heights at which the mercury stands at the surface of the earth and on the summit of the mountain.*

II. *Multiply this difference by the constant number, 63946—the result is the elevation in feet, if the mean temperature of the surface of the earth and the elevation is 69·68° Fhr.*

III. *If the mean temperature of the two elevations be not 69·68° Fhr., add $\frac{1}{480}$ of the whole weight found for each degree above 69·68°, or subtract the same quantity if the mean temperature be below.*

EXAMPLE 282.—Humboldt found that at the level of the sea, near the foot of Chimborazo, the mercury stood at the height of 30 inches, while at the summit of the mountain it was only 14·85 inches. At the same time the temperature at the base of the monntain was 87° Fahr., and at the top 50·40° Fahr. What is the height of Chimborazo?

SOLUTION.

Log. of 30 = 1·477121, log. of 14·85 = 1·171724 and mean temperature =

$$\frac{87° + 50·4°}{2} = \frac{137·4°}{2} = 68·70.°$$

Then 1.477121 − 1·171724 = ·305397.
And ·305397 × 63946 = 19539 feet.

Since the mean temperature of the two stations is 1 less than 69.68°, we deduct $\frac{1}{480}$ of the elevation found.

$\frac{1}{480}$ of 19539 = 40.7 ft. and 19539 − 40·7 = 19498.3 ft. *Ans.*

LESLIE'S RULE.

FOR MEASURING HEIGHTS BY THE BAROMETER WITHOUT THE USE OF LOGARITHMS.

I. *Note the exact height of the column of mercury at the base and at the summit of the elevation.*

II. *Then say, as the sum of the two pressures is to their difference, so is the constant number* 52000 *to the answer in feet.*

EXAMPLE 283.—The barometer in a balloon is observed to stand at a height of 22 inches, while at the surface of the earth it stands at 29·8 inches; what is the elevation of the balloon?

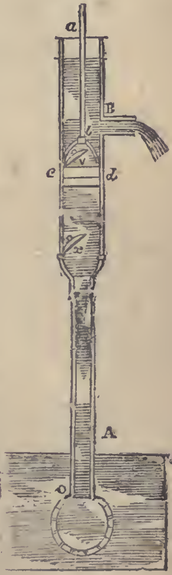
SOLUTION.

$$22 + 29{\cdot}8 : 29{\cdot}8 - 22 : : 52000 : Ans$$

$$\text{Or, } 51{\cdot}8 : 7{\cdot}8 : : 52000 : \frac{52000 \times 0{\cdot}1}{51{\cdot}8} = 7830{\cdot}1 \text{ ft.. } Ans.$$

EXERCISE.

Fig. 22.



284. At what height would the mercury stand in the barometer at an elevation of 29·7 miles above the earth's surface? *Ans.* 0.0146 inches.

NOTE.—Divide 29 7 by 2·7 (See Art. 212,) the quotient is 11, then divide 30 inches by 2^{11}, i. e. 2048, and the result is the answer.

285. At what height will the barometer stand in a balloon which is at an elevation of $16\frac{1}{5}$ miles? *Ans.* ·46875 inches.

286. *It is observed that while the barometer at the base of a mountain stands at a height of 30 inches, at the top of the mountain it stands at a height of only 18 inches, required the height of the mountain? *Ans.* 13000 feet.

287. *While the mercury at the base of a mountain stands at the height of 29·5 inches, at the summit of the mountain the barometer indicates a pressure of only 20·4 inches, what is the height of the mountain? *Ans.* 9482·9 feet.

288. †While in a balloon the barometer indicates a pressure of only 19 inches, at the surface of the earth

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* Use Leslie's rule.

† Use Halley's rule with correction for temperature; i. e., the second of the rules given.

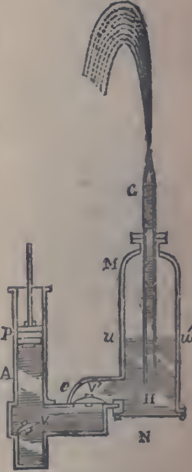
the pressure is 29·94 inches—taking the mean temperature of the two stations at 72·50°, what is the elevation of the balloon?
Ans. 12703 feet.

236. The *common pump* consists of a barrel *SB*, a tube *AS*, which descends into the water reservoir, a piston *cd*, moving air-tight in the barrel and two valves, *v* and *x*, which act in the same manner as in the exhausting syringe of the air pump.

Note 1,—When the machine begins to act the piston is raised and produces a vacuum below it in the barrel, and the atmospheric pressure on the water in the reservoir forces it up the tube and through the valve *x* into the lower part of the barrel. As the piston descends the valve *x* closes and the water obtained in the barrel passes through the valve *v* above the piston to be *lifted* out at the next stroke. Hence the common pump is sometimes called a *lifting pump*.

Note 2.—Since the specific gravity of mercury is 13·596 and the pressure of the atmosphere sustains a column of mercury, 30 inches in height—it follows that atmospheric pressure will sustain a column of water 30 × 13·596 inches, or 34 ft. in height. Hence the vertical distance of the valve *x* above the surface of the water in the reservoir must be less than 34 feet, or taking the variations in atmospheric pressure into account, about 32 feet.

Fig. 23.

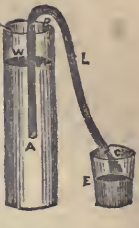


237. The *forcing pump* consists of a suction pump *A*, in which the piston *P* is a solid plug without a valve. When the piston *P* descends the valve *v* closes and the water is forced through the valve *v'* into the chamber *MN*. The upper part of this chamber is filled with compressed air, which, by the pressure it exerts against the surface of the water, *ww'* drives it with considerable force through the pipe or tube *HG*.

Note.—Sometimes the forcing pump is used without the air chamber, *MN*. Fig 23 exhibits the arrangement of the valves, &c., in a common fire engine with the exception that there is another similar forcing pump on the other side of the air chamber. *HG* represents the tube leading to the hose.

238. The *Syphon* is a bent tube of glass or other material having one leg somewhat longer than the other, and is used for transferring liquids from one vessel to another.

Fig. 24.



NOTE.—The machine is set in operation by immersing the shorter leg in the liquid to be decanted, and sucking the air out of the tube, when the pressure of the atmosphere forces the liquid into the syphon over the bend and down through the longer leg. Instead of sucking the air out of the syphon, the instrument may be set in operation by first filling it with the liquid, and, while thus full, placing the finger over each end, and immersing the shorter leg in the liquid.

NOTE 2.—In order to understand why one limb must be shorter than the other, it is only necessary to remember that the pressure of the atmosphere acts as much at one extremity as at the other. If we raise the column of liquid as far as *B*, by sucking at the extremity *C*, and then withdraw the mouth, the water falls back into the vessel *F*. The column will likewise run *back* if we get it no farther than *L*, which is the level of the water in the vessel *F*, because at that point the *upward* pressure of the atmosphere prevails over the *downward* pressure of the liquid, but if we get the column below *L*, the downward pressure of the liquid exceeds the upward pressure of the atmosphere, and the liquid will flow.

Thus the motion of the fluid in the syphon is similar to the motion of a chain hanging over a pulley,—if the two parts of the chain be equal, the fluid remains at rest, but if one end be longer than the other, it moves in the direction of the longer; and fresh links, so to speak, are added continuously to the fluid chain by the atmospheric pressure exerted on the surface of the water.

CHAPTER VII.

DYNAMICS.

239. When the forces which are the subject of investigation are *balanced*, the consideration of them properly comes under the science of *Statics ;* but when they cease to be balanced, and the body acted upon is *set in motion*, other principles become involved, and the investigation of these constitutes the more complex science of *Dynamics*.

240. *Statics* is a *deductive science*, since all its facts are deducible, like those of Arithmetic and Geometry, from abstract truths; *dynamics* is an *inductive, experimental, physical science*, many of its principles being capable of proof only by an appeal to the laws of nature.

241. *Force* may be defined to be the *cause* of the *change of motion*, i.e., force is required :—

1st. To change the state of a body from rest to motion, or from motion to rest.

2nd. To change the velocity of motion.

3rd. To change the direction of motion.

242. Forces are either *instantaneous* or *continued*, and *continued* forces are either *accelerating*, *constant*, or *retarding*.

243. *Motion* may be defined to be the opposite of rest, or a continuous changing of place.

244. *Motion* has two qualities. *direction* and *velocity*, and is of three kinds—

1st. *Direct;*

2nd. *Rotatory* or *Circular;* and

3rd. *Vibratory* or *Oscillatory.*

245. An *accelerating*, *constant*, or *retarding* force produces an *accelerated*, *uniform* or *retarded* motion.

246. *Velocity* is the degree of speed in the motion of a body, and may be either *uniform* or *varied*. It is *uniform* when all equal spaces, great or small, are passed over in equal times.

247. The principles of the composition and resolution of *force* are equally applicable to *motion*.

248. *Momentum*, or *Motal Force* or *Quantity* of *Motion*, is the force exerted by a mass of matter in motion.

249. The *momenta* of a body depends upon its weight and velocity, thus:

I. When the velocities of two moving bodies are equal, their momenta are proportional to their masses.

II. When the masses of two moving bodies are equal, their momenta are proportional to their velocities.

III. When neither the masses nor velocities of two moving bodies are equal, their momenta are in proportion to the products of their weights by their velocities.

NOTE.—When we speak of multiplying a velocity by a weight, we refer to multiplying the *number of units* of weight by the *number of units* of velocity, and it makes no difference what units of each kind are employed, for the product, thus obtained, means nothing by itself, but only by comparison with other products similarly obtained by the use of the same units.

For example, when we say that a weight of 11 lbs. moving 6 feet per second, has a momentum of 66, all we mean is, that in this case the weight strikes a body at rest with 66 times the force that a body weighing one lb. and moving only one foot per second would exert.

250. If a moving body M, having a velocity V, strike another m at rest, so that the two masses shall coalesce, and move on together with a velocity v, then $M \times V =$

$(M + m) + v$; or whatever momentum may be acquired by the body m must be lost by M.

251. If a moving body M, having a velocity V, strike another body m, moving in the same direction, with a velocity v, so that the two may coalesce, and move on together with a velocity *vel*,—then $M \times V + m \times v = (M + m) \times vel$, or in other words the two bodies united have the same momentum that they separately had before impact.

252. If a moving body M, having a velocity V, strike another body m moving with a velocity v, in the opposite direction, so that the two masses shall coalesce and move on together with a velocity *vel*,—then $M + V \sim m \times v = (M + m) \times vel$, or in other words, the body moving with least force will destroy as much of the momentum of the other, as is equal to its own momentum.

253. If a moving body M, having a velocity V, strike another body m moving obliquely towards it with a velocity v, so that the two masses shall coalesce and move on together, then by representing their momenta, just before impact, by lines in the direction of their motion and completing the parallelogram, the diagonal will represent the quantity and direction of the momentum of the combined mass.

EXAMPLE 289.—What is the momentum of a body weighing 78 lbs., and moving with a velocity of 20 feet per second.

SOLUTION.

Momentum$=78 \times 20 = 1560$. *Ans.*

That is, the momentum of such a body is 1560 times as great as the momentum of a body weighing only 1 lb., and moving only 1 ft. per second.

EXAMPLE 290.—If a body weighing 67 lbs. be moving with the velocity of 11 feet per second, and strike a second body at rest weighing 33 lbs., so that the two bodies may coalesce, and move on together, what will be the velocity of the united mass?

SOLUTION.

Art. 250.—If M be the moving body, V its velocity, m the body at rest, and v the velocity of the united mass:—

Then $(M + m) \times v = M \times V$ and therefore $v = \frac{M \times V}{M + m}$

In this example, $M = 67$, $V = 11$, and $m = 33$.

Then $v = \frac{M \times V}{M + m} = \frac{67 \times 11}{67 + 33} = \frac{737}{100} = 7{\cdot}37$ feet per second. *Ans.*

EXAMPLE 291.—If a body weighing 50 lbs., and moving with a velocity of 100 ft. per second, come in contact with another body weighing 40 lbs., and moving in the same direction with a velocity of 20 feet per second, so that the two bodies coalesce and move on together, what will be the velocity and momentum of the united mass?

SOLUTION.

ART. 251.—If M and m be the two bodies, and V and v their separate velocities, and *vel* the velocity of the united mass;—

Then $(M + m) \times vel = M \times V + m \times v$. Hence $vel = \frac{M \times V + m \times V}{M + m}$

In this example $M = 50$, $m = 40$, $V = 100$ and $v = 20$.

Then $Vel = \frac{M \times V + m \times v}{M + m} = \frac{50 \times 100 + 40 \times 20}{50 + 40} = \frac{5000 + 800}{90} = \frac{5800}{90}$

$= 64\frac{4}{9}$ ft. per second, and momentum $= (50 + 40) \times 64\frac{4}{9} = 5800$. *Ans.*

EXAMPLE 292.—If a body weighing 120 lbs., and moving to the east with a velocity of 40 feet per second, come into contact with a second body weighing 90 lbs., and moving to the west, with a speed of 80 feet per second, so that the two bodies coalesce and move onward together, in what direction will they move, with what velocity, and what will be their momentum?

SOLUTION.

From Art. 252, if M and m be the bodies, and V and v their respective velocities, and *vel* the velocity of the united mass after impact:—

Then $(M + m) \times vel = M \times V \sim m \times v$ and hence

$$vel = \frac{M \times V \sim m \times v}{M + m}$$

In this example $M = 120$, $m = 90$, $V = 40$ and $v = 80$.

Then $vel = \frac{M \times V \sim m \times v}{M \times m} = \frac{(120 \times 40) \sim (90 \times 80)}{120 + 90} = \frac{4800 \sim 7200}{210}$

$= \frac{2400}{210} = 11\frac{3}{7}$ feet per second = the velocity. $11\frac{3}{7} \times (120+90) = 11\frac{3}{7} \times 210 = 2400$ = momentum.

And since 90×80, the momentum of the body moving to the west is greater than 120×40, the momentum of the body moving to the east, the united mass moves to the west.

EXERCISE.

293. What is the momentum of a body weighing 79 lbs., moving with a velocity of 64 feet per second? *Ans.* 5056.

294. Which would strike an object with the greatest force, a bullet weighing one ounce and propelled with a velocity of 2000 feet per second, or a ball weighing 5 lbs., and thrown with a velocity of 28 feet per second?

Ans. Momentum of bullet = 125.
" of ball = 140.
Therefore the ball would exert most force of impact.

295. Which has the greatest momentum, a train of cars weighing 170 tons and moving at the rate of 40 miles per hour, or a steamer weighing 790 tons and moving at the rate of 9 miles per hour? *Ans.* Momentum of train = 6800, of steamer = 7110, and therefore the latter has most momentum.

296. If a body weighing 60 lbs. and moving at the rate of 86 feet per second, come in contact with another body weighing 400 lbs., and moving in the same direction at the rate of 12 feet per second, so that the two bodies coalesce and move on together; what will be the velocity and momentum of the united mass?
Ans. Velocity = $21\frac{15}{23}$ feet per second; momentum = 9960.

297. If a body weighing 56 lbs. and moving with a velocity of 80 feet per second come in contact with a body at rest, weighing 70 lbs., so that the two bodies coalesce and move on together; what will be the velocity of the united mass? *Ans.* $35\frac{5}{9}$ feet per second.

298. If a body weighing 77 lbs. and moving from south to north, with a velocity of 40 feet per second, come in contact with another body weighing 220 lbs. and moving from north to south, with a velocity of 14 feet per second, so that the two bodies coalesce; in what direction and with what velocity does the united mass move?
Ans. Their momenta exactly neutralize each other and the bodies come to a state of rest.

299. If a body weighing 70 lbs., moving to the south with a velocity of 70 feet per second, come in contact with another body which weighs 80 lbs. and is moving to the north with a velocity of 60 feet per second, so that the two bodies coalesce and move on together; in what direction will they move and with what velocity and momentum? *Ans.* To the south with velocity of 8 inches per second. Momentum of united mass = 100.

300. If a body weighing 600 lbs. and moving to the west with a velocity of 40 feet per second, come in contact with a second body weighing 50 lbs. and moving to the east with a velocity of 20 feet per second, and after the two have coalesced they come in contact with a third body which weighs 100 lbs., and is moving in an opposite direction with a velocity of 150 feet per second, and the three then coalesce and move on together; in what direction will their motion be, and what will be the velocity and momentum of the united mass? *Ans.* Direction, west.
Velocity = $10\frac{2}{3}$ feet.
Momentum = 8000.

254. When force is communicated by impact to a body at rest, the body will remain at rest until the force is *distributed* throughout all the atoms of the mass, unless a fragment be broken off by the force of impact, in which case this fragment alone moves.

LAWS OF MOTION.

255. THE FIRST LAW OF MOTION.—*Every body must persevere in a state of rest or of uniform motion in a straight line, unless it be compelled to change that state by force impressed upon it.*

256. THE SECOND LAW OF MOTION.—*Every change of motion must be in proportion to the impressed force, and must be in the direction of that straight line in which the impressed force acts.*

257. THIRD LAW OF MOTION.—*All action is attended by a corresponding re-action, which is equal to it in force and opposite in direction.*

These laws are commonly known as Sir I. Newton's laws of motion—in reality however the first is due to Kepler, the second to Newton, and the third to Galileo.

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258. When a moving elastic body strikes against the surface of another body, the direction of its motion is changed, and the motion thus resulting is said to be reflected. Here :—

1st. The angle at which the moving body strikes the surface of the other is called the *angle of incidence ;*

2nd. The angle at which the moving body rebounds is called the *angle of reflection;* and

3rd. The angle of reflection is always equal to the angle of incidence.

259. In a vacuum, all bodies, whatever may be their form or density, fall towards the centre of the earth in vertical lines and with equal rapidity ; but in ordinary circumstances, i. e., falling through the air, only heavy bodies fall in vertical lines, and the density and form of a body materially affect its velocity.

260. The resistance which a body encounters in moving through the atmosphere or any other fluid, varies :—

1st. As the surface of the moving body.

2nd. As the square of the velocity of the moving body. (See Art. 147.)

NOTE.—In the case of heavy bodies falling through the air, the resistance of the atmosphere produces a considerable discrepancy between the actual fall of bodies and the distance through which they should theoretically fall. Thus, it has been found by experiment that a ball of lead dropped from the lantern of St. Paul's Cathedral required $4\frac{1}{2}$ seconds to reach the pavement, a distance of 272 feet. But in $4\frac{1}{2}$ seconds the ball ought to have fallen 324 feet by theory, the difference of 52 feet being due to the retarding force of the atmosphere.

261. A heavy body falling from a height moves with a uniformly accelerated motion, since the attraction of gravity which causes the descent of the body never ceases to act, and the falling body gains at each moment of its descent a new impulse, and thus an increase of velocity, so that its final velocity is the sum of all the infinitely small but equal increments of velocity thus communicated.

262. *Hence the velocity of a falling body at the end of the second moment of its descent is* TWICE *that which it had at the end of the first second; at the end of the third second,* THREE TIMES *that which it had at the end of the first; at the end of the fourth,* FOUR *times, &c.*

263. *Hence also a heavy body starting from a state of rest and falling during any time, acquires a velocity, which would in the same space of time carry it through twice the space it has passed over.*

264. It has been ascertained by numerous and careful experiments, that a falling body acquires at the end of the first second of its descent, a velocity equal to that of $32\frac{1}{6}$ feet per second, and hence during the first second of its descent a body falls through one-half of $32\frac{1}{6}$ feet, i. e., through $16\frac{1}{12}$ feet.

NOTE 1.—The average speed of the falling body is the arithmetical mean between its initial and terminal velocities, or in the case of the first second of its fall, between 0 and $32\frac{1}{6}$, and this is $16\frac{1}{12}$.

NOTE 2.—In the following exercises we shall use 32 and 16 in place of $32\frac{1}{6}$ and $16\frac{1}{12}$, since the fractions materially increase the labor of making the calculations without illustrating the principles any better than the whole numbers used alone.

265. ANALYSIS OF THE MOTION OF A FALLING BODY.

NUMBER OF SECONDS.	SPACE PASSED OVER EACH SECOND.	TERMINAL VELOCITIES.	TOTAL SPACE.
1	1	2	1
2	3	4	4
3	5	6	9
4	7	8	16
5	9	10	25
6	11	12	36
7	13	14	49
8	15	16	64
9	17	18	81
10	19	20	100

NOTE.—The numbers in the second, third and fourth columns mean *so many times* 16 feet.

From this it is evident that :—

I. *The spaces through which the body descends in equally successive portions of time, increase as the odd numbers,* 1, 3, 5, 7, 9, *&c., and hence the space through which the body falls during any second of its flight, is found by multiplying* 16 *feet by the odd number which corresponds to that second ; i. e., one less than twice the number of the second.*

II. *The final velocity acquired by a falling body at the end of successive equal portions of time, varies as the even numbers,* 2, 4, 6, 8, *&c., and hence the final velocity acquired by a body at the end of any second of its fall, is found by multiplying* 16 *feet by twice the number of seconds.*

III. *The whole space passed over by a body falling during equal successive portions of time, varies as the square of the numbers,* 1, 2, 3, 4, *&c., and hence the whole space passed over during any given number of seconds, is found by multiplying* 16 *feet by the square of the number of seconds.*

266. Let $t =$ the time of descent in seconds, $v =$ the terminal velocity, i. e., the velocity acquired at the end of the last second of its fall, $s =$ whole space passed over, and $g = 32$, i, e., the measure of the attraction of gravity.

Then Art. 263, the time is equal to the space divided by half the terminal velocity, or $t = \frac{s}{\frac{1}{2}v} = \frac{2s}{v}$

Again (Art. 265, III) the whole space passed over is equal to 16, i. e., half of the gravity, g, multiplied by the square of the time or $s = \frac{1}{2}gt^2$.

Also (Art. 265, I) the terminal velocity is equal to 16, i. e., $\frac{1}{2}g$ multiplied by twice the time or $v = \frac{1}{2}g \times 2t = gt$.

These three formulas, viz: $s = \frac{1}{2}gt^2$, $v = gt$ and $t = \frac{2s}{v}$ are fundamental, and the remaining six of the following table are derived from them by transposition and substitution:—

TABLE OF FORMULAS FOR DESCENT OF BODIES FALLING FREELY THROUGH SPACE.

NO.	GIVEN.	TO FIND.	FORMULAS.	WHENCE DERIVED.
I	t, g		$s = \frac{1}{2}gt^2$	*Art.* 265, *III.*
II	v, g	s	$s = \frac{v^2}{2g}$	*From formula V.*
III	t, v		$s = \frac{1}{2}tv$	*From formula VII.*
IV	g, t		$v = gt.$	*Art.* 265, *I.*
V	g, s	v	$v = \sqrt{2gs}.$	*From IV and VII by substituting the value of t.*
VI	s, t		$v = \frac{2s}{t}$	*From formula VII.*
VII	s, v		$t = \frac{}{v}$	*Art.* 263.
VIII	v, g	t	$t = \frac{v}{g}$	*From formula IV.*
IX	s, g		$t = \sqrt{\frac{2s}{g}}$	*From formula I.*

267. When a body is thrown vertically upward it rises with a regularly retarded motion, losing 32 feet of its original velocity every second, and it occupies as much time in rising as it would have required in falling to acquire its initial velocity.

268. If a body be projected upwards or downwards with a given initial velocity V, and is at the same time acted upon by the force of gravity, then when the body descends, in t seconds the initial velocity alone would carry it through Vt feet, and gravity alone would carry it through $\frac{1}{2}gt$ feet, therefore together they carry it through $Vt + \frac{1}{2}gt^2$ feet, and the terminal velocity will evidently be $V + tg$.

When the body ascends the initial velocity acting alone would carry it in t seconds through Vt feet, but in t seconds the force of gravity would draw it downward through $\frac{1}{2}gt^2$ feet, and therefore its whole ascent will be $Vt-\frac{1}{2}gt^2$, and its terminal velocity will be $V-gt$. Hence,

(X) $s = Vt + \frac{1}{2}gt^2$ *when the body descends.*

(XI) $s = Vt - \frac{1}{2}gt^2$ *when the body ascends.*

(XII) $v = V + tg$ *when the body descends.*

(XIII) $v = V - tg$ *when the body ascends.*

EXAMPLE 301.—Through how many feet will a body fall during the 11th second of its descent?

SOLUTION.

From Art. 265, I. space $= \left\{ (11 \times 2)-1 \right\} \times 16 = (22-1) \times 16 = 21 \times 16 = 336$ feet. *Ans.*

EXAMPLE 302.—Through how many feet will a body fall during the 17th, the 43rd, and the 61st second of its descent?

SOLUTION.

For the 17th second $17 \times 2 = 34 - 1 = 33 \times 16 = 528$ feet. *Ans.*
For the 43rd " $43 \times 2 = 86 - 1 = 85 \times 16 = 1360$ feet. *Ans.*
For the 61st " $61 \times 2 = 122 - 1 = 121 \times 16 = 1936$ feet. *Ans.*

EXAMPLE 303.—What will be the terminal velocity of a falling body at the end of the 9th second of its descent?

SOLUTION.

Formula IV. $v = gt = 32 \times 9 = 288$ feet per second. *Ans.*

EXAMPLE 304.—What will be the terminal velocity of a falling body at the end of the 25th second of its fall, also at the end of the 33rd second?

SOLUTION.

Formula IV. $v = gt = 32 \times 25 = 800$ feet per second at end of 25th second.
$v = gt = 32 \times 33 = 1056$ feet per second at end of 33rd "

EXAMPLE 305.—Through how many feet will a body fall during 5 seconds?

SOLUTION.

Formula I. $s = \frac{1}{2}gt^2 = \frac{1}{2} \times 32 \times 5^2 = 16 \times 25 = 400$ feet. *Ans.*

EXAMPLE 306.—Through how many feet will a body fall in 12 seconds?

SOLUTION.

Formula I. $s = \frac{1}{2}gt^2 = \frac{1}{2} \times 32 \times 12^2 = 16 \times 144 = 2304$ feet. *Ans.*

EXAMPLE 307.—If a body has fallen until it has acquired a terminal velocity of 400 feet per second, what is the whole space through which it has descended?

SOLUTION.

Formula II. $s = \frac{v^2}{2g} = \frac{400^2}{2 \times 32} = \frac{160000}{64} = 2500$ feet. *Ans*

EXAMPLE 308.—How long must a body fall in order to acquire a terminal velocity of 1000 feet?

SOLUTION.

Formula VIII. $t = \frac{v}{g} = \frac{1000}{32} = 31\frac{1}{4}$ seconds. *Ans.*

EXAMPLE 309.—How long must a body fall in order to acquire a terminal velocity of 8000 feet per second?

SOLUTION.

Formula VIII. $t = \frac{v}{g} = \frac{8000}{32} = 250$ seconds. *Ans.*

EXAMPLE 310.—What time does a body require to fall through 11200 feet?

SOLUTION.

Formula IX. $tt = \sqrt{\frac{2s}{g}} = \sqrt{\frac{2 \times 11200}{32}} = \sqrt{700} = 26{\cdot}45$ seconds. *Ans.*

EXAMPLE 311.—When a body has descended through 4400 feet, what velocity has it acquired?

SOLUTION.

Formula V. $v = \sqrt{2gs} = \sqrt{2+32 \times 4400} = \sqrt{281600} = 530{\cdot}6$ ft. per second.

EXAMPLE 312.—If an arrow be shot vertically upwards and reach the ground again after the lapse of 20 seconds, to what height did it rise?

SOLUTION.

From Art. 267 it appears that the arrow will be as long ascending as descending, and hence the problem is reduced to finding the distance through which the arrow will fall in half of 20 seconds, i. e., in 10 seconds.

Then formula I. $s = \frac{1}{2}gt^2 = \frac{1}{2} \times 32 \times 10^2 = 16 \times 100 = 1600$ feet. *Ans.*

EXAMPLE 313.—If a cannon ball be fired vertically with an initial velocity of 1600 feet per second, to what height will it rise?

SOLUTION.

First, the time it ascends is equal to the time it would require if descending to acquire a terminal velocity of 1600 feet.

By formula VIII. $t = \frac{v}{g} = \frac{1600}{32} = 50$ seconds = time of ascent.

Then formula XI. $s = Vt - \frac{1}{2}gt^2 = 1600 \times 50 - \frac{1}{2} \times 32 \times 50^2 = 80000 - 16 \times 2500 = 80000 - 40000 = 40000$ feet *Ans.*

EXAMPLE 314.—If a body be shot upward with an initial velocity of 1200 feet per second, at what height will it be at the end of the 10th second, and also at the end of the 70th second of its flight?

SOLUTION.

Formula XI. $s = Vt - \frac{1}{2}gt^2 = 1200 \times 10 - \frac{1}{2} \times 32 \times 10^2 = 1200 - 1600 = 10400$ feet = elevation at end of 10th second.

Also $1200 \times 70 - \frac{1}{2} \times 32 \times 70^2 = 84000 - 16 \times 4900 = 84000 - 784000 = 560$ feet = elevation at end of the 70th second.

EXAMPLE 315.—If a cannon ball be fired vertically with an initial velocity of 2400 feet per second :—

1st. In how many seconds will it again reach the ground?
2nd. How far will it rise?
3rd. Where will it be at the end of the 40th second?
4th. What will be its terminal velocity?
5th. In what other moment of its flight will it have the same velocity as at the end of the 19th second of its ascent?

SOLUTION.

Since the initial velocity = terminal velocity = 2400 feet.

1. Formula VIII. time of ascent $= \frac{v}{g} = \frac{2400}{32} = 75$ seconds, and since it is as long ascending as descending, it again reaches the ground in 150 sec.

II. Formula I. $s = \frac{1}{2}gt^2 = \frac{1}{2} \times 32 \times 75^2 = 16 \times 5625 = 90000$ ft. = height to which it rises.

III. Formula XI. $s = Vt - \frac{1}{2}gt^2 = 2400 \times 40 - \frac{1}{2} \times 32 \times 40^2 = 96000 - 16 \times 1600 = 96000 - 25600 = 70400$ ft. = elevation at end of 40th second.

IV. Terminal velocity = initial velocity = 2400 feet per second.

V. Since the whole time of flight = 150 seconds, and, since at all equal spaces of time from the moment it ceases to ascend and begins to descend, the velocity is the same in rising as in falling, it follows that the moment in which the body has the same velocity as at the end of the 19th second of its ascent is 19 full seconds before it again reaches the ground, or in $150 - 19 = 131$st second, i. e., in the end of the 131st second.

EXAMPLE 316.—If a body is thrown downwards from an elevation with an initial velocity of 70 feet per second, how far will it descend in 27 seconds?

SOLUTION.

Formula X. $s = Vt + \frac{1}{2}gt^2 = 70 \times 27 + \frac{1}{2} \times 32 \times 27^2 = 1890 + 16 \times 729 = 1890 + 11664 = 13554$ ft. *Ans.*

EXAMPLE 317.—If a body is thrown down from an elevation with an initial velocity of 140 feet per second, what will be its velocity at the end of the 30th second?

SOLUTION.

$v = V + tg = 140 + 30 \times 32 = 140 + 960 = 1100$ feet per second. *Ans.*

EXAMPLE 318.—If a body be projected vertically with an initial velocity of 400 feet per second, what will be its velocity at the end of the 12th second?

SOLUTION.

Formula XIII. $v = V - tg = 400 - 12 \times 32 = 400 - 384 = 16$ feet per second. *Ans.*

EXAMPLE 319.—If a cannon ball be fired vertically upwards with an initial velocity of 1800 feet per second :—

1st. In how many seconds will it again reach the ground?
2nd. What will be its terminal velocity?
3rd. How far will it rise?
4th. Where will it be at the end of the 90th second?
5th. In what other moment of its flight will it have the same velocity as at the end of the 27th second of its ascent?

SOLUTION.

I. $t = \frac{v}{g} = \frac{1800}{32} = 56\frac{1}{4}$ = time of ascent or descent, hence whole time of flight $= 56\frac{1}{4} \times 2 = 112\frac{1}{2}$ seconds.

II. Terminal velocity = initial velocity = 1800 feet per second.

III. Formula 1. $S = \frac{1}{2}gt^2 = \frac{1}{2} \times 32 \times (56\frac{1}{4})^2 = 16 \times 3164 \cdot 0625 = 50625$ ft.

IV. Formula XI. $S = Vt - \frac{1}{2}gt^2 = 1800 \times 90 - \frac{1}{2} \times 32 \times 90^2 = 162000 - 16 \times 8100 = 162000 - 129600 = 32400$ ft. = elevation at end of the 90th second.

V. $112\frac{1}{2} - 27 = 85\frac{1}{2}$ = middle of 86th second of flight.

EXAMPLE 320.—A stone is dropt into the shaft of a mine and is *heard* to strike the bottom in 9 seconds; allowing sound to travel at the rate of 1142 ft. per second, and taking $g = 32\frac{1}{6}$; required the depth of the shaft.

SOLUTION.

Let x = time stone takes to fall. Then $(9-x)$ = time sound takes to reach the top and $x^2 \times 16\frac{1}{12}$ = depth of shaft $= (9-x) \times 1142$ feet.

Therefore $\frac{193x^2}{12} = 1028 - 1142x$.

$193x^2 + 13704x = 123336.$

$148996x^2 + 10579488x + 187799616 = 95215392 + 187799616 = 283015008.$

$386x + 13704 = 16823 +$

$386x = 3119$

$x = 8 \cdot 0803$ = number of seconds body was falling.

$9 - x = 9 - 8 \cdot 0803 = \cdot 9197$ = time sound travelled.

And $1142 \times \cdot 9197 = 1050 \cdot 2974$ feet = depth of shaft.

EXAMPLE 321.—A body has fallen through m feet when another body begins to fall at a point n feet below it; required the distance the latter body will fall before it is passed by the former?

FIRST SOLUTION.

At end of m ft. $t = \sqrt{\frac{2s}{g}} = \sqrt{\frac{2m}{g}}$, and $v = gt = g\sqrt{2m} = \sqrt{2mg}$, and since

n = distance to be traversed $t = \frac{n}{\sqrt{2mg}}$, hence $S = \frac{1}{2}gt^2 = \frac{1}{2}g \times \left(\frac{n}{\sqrt{2mg}}\right)^2 = \frac{1}{2}g \times \frac{n^2}{2mg} = \frac{n^2}{4m}$. *Ans.*

SECOND SOLUTION.

Let x = distance. Then (of 2nd = body) $t = \sqrt{\frac{2S}{g}} = \sqrt{\frac{2r}{g}}$ and $\sqrt{\frac{(2m + x + x)}{g}}$ = entire time taken by the first body to pass through whole space.

Then $\sqrt{\frac{2(m+n+x)}{g}} - \sqrt{\frac{2m}{g}} = \sqrt{\frac{2x}{g}}$ and multiplying all by $\sqrt{g}$.

$$\sqrt{2(m+n+x)} - \sqrt{2m} = \sqrt{2x}.$$
$$\sqrt{2(m+n+x)} = \sqrt{2x} + \sqrt{2m}, \text{ and squaring.}$$
$$2(m+n+x) = 2x + 2m + 2\sqrt{4mx}.$$
$$2m + 2n + 2x = 2x + 2m + 2\sqrt{4mx}.$$
$$2n = 4\sqrt{mx}.$$
$$n = 2\sqrt{mx}.$$
$$n^2 = 4mx.$$
$$\therefore x = \frac{n^2}{4m}. \quad Ans.$$

*EXERCISE.

322. Through how many feet will a body fall during the 37th second of its descent? *Ans.* 1168 ft.

323. Through what space will a body descend in 25 seconds? *Ans.* 10000 ft.

324. With what velocity does a body move at the close of the 20th second of its fall? *Ans.* 640 ft. per sec.

325. During how many seconds must a body fall in order to acquire a terminal velocity of 1100 ft. per sec.? *Ans.* 34⅜ sec.

326. Through what space must a falling body pass before it acquires a terminal velocity of 1700 ft. per sec.? *Ans.* 45156¼ ft.

327. What will be the terminal velocity of a body that has fallen through 25000 ft.? *Ans.* 1264·8 ft.

328. If a body is projected upwards with an initial velocity of 6000 ft. per second, where will it be at the end of the 40th second? *Ans.* At an elevation of 214400 ft.

329. If a body be thrown downward with an initial velocity of 120 ft. per second, through how many feet will it fall in 32 seconds? *Ans.* 20224 ft.

330. A cannon ball is fired vertically, with an initial velocity of 1936 per second:—

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* In all cases, when not otherwise directed, use $g = 32$ ft.

1st. How far will it rise?
2nd. Where will it be at the end of the 6th second?
3rd. In how many seconds will it again reach the ground?
4th. What will be its terminal velocity?
5th. In what other moment of its flight will it have the same velocity as at the end of the 13th second of its ascent?

Ans. 1st. 58564 ft.
2nd. At an elevation of 11040 ft.
3rd. 121 seconds.
4th. 1936 ft. per second.
5th. At end of 108th sec. of flight.

331. If a body be projected vertically with an initial velocity of 4000 feet per second, taking gravity to $32\frac{1}{6}$ feet:—
1st. How high will the body rise?
2nd. Where will it be at the end of the 50th second?
3rd. Where will it be at the end of the 100th second?
4th. Where will it be at the end of the 200th second?
5th. In what time will it again reach the ground?

Ans. 1st. 248704·66 ft.

2nd.	At an elevation of	159791·66 ft.
3rd.	"	239166·66 ft.
4th.	"	156666·66 ft.
5th.	248·70 seconds.	

332. If a cannon ball be fired vertically with an initial velocity of 1100 feet per second, what will be its velocity at the end of the 7th second, at the end of the 20th second, and at the end of the 33rd second?

Ans. End of the 7th sec. vel. = 876 ft.
" 20th " = 460 ft.
" 33rd " = 44 ft.

333. If a stone be dropped into a well and is seen to strike the water after the lapse of 5 seconds, how deep is the well?

Ans. 400 ft.

334. If a stone be thrown downwards with an initial velocity of 250 ft. per second, what will be its velocity at the end of the 3rd, the 9th, the 30th and the 90th seconds of its descent?

Ans. End of 3rd sec. vel. = 346 ft. per sec.
" 9th " = 538 ft. "
" 30th " = 1210 ft. "
" 90th " = 3130 ft. "

335. A stone is dropt into the shaft of a mine and is *heard* to strike the bottom in 12·76 seconds, assuming that sound travels at the rate of 1100 ft. per second, what is the depth of the mine? *Ans.* 1936 ft.

336. A body has fallen through 400 feet, when another body begins to fall at a point 2500 feet below it; through what space will the latter body fall before the former overtakes it? *Ans.* 3906¼ feet.

337. A body *A* has fallen during *m* seconds, when another body *B* begins to fall, *f* feet below it; in what time will *A* overtake *B*?

Ans. $\dfrac{f}{32m}$

DESCENT ON INCLINED PLANES.

269. When a body is descending an inclined plane, a portion of the gravity of the body is expended in pressure on the plane and the remainder in accelerating the motion of the descending body.

270. The following are the laws of the descent of bodies on inclined planes:

I. The pressure on the inclined plane is to the weight of the body as the base of the plane is to its length.

II. The terminal velocity of the descending body is that which it would have acquired in falling freely through a distance equal to the height of the plane.

III. The space passed through by a body falling freely is to that gone over an inclined plane, in equal times, as the length of the plane is to its height.

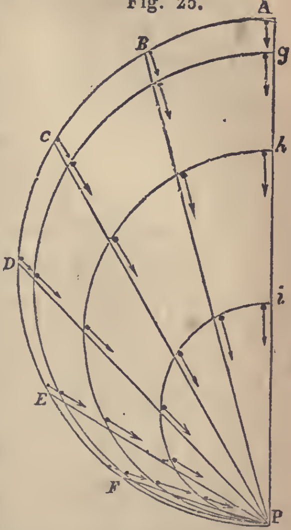
IV. If a body which has descended an inclined plane meets at the foot of it another inclined plane of equal altitude, it will ascend this plane with the velocity acquired in coming down the former, it will then descend the second and re-ascend the former plane, and will thus continue oscillating down one plane and up the other.

NOTE.—The same takes place if the motion be made in a curve instead of on an inclined plane. In practice, however, the resistance of the atmosphere and friction retard the motion very greatly at each oscillation and very soon bring the body to a state of rest.

271. The final velocity, neglecting friction, on arriving at the bottom of the plane, is dependent solely on the height of the plane, and will be the same for all planes of equal height, however various may be their lengths; and the times of descent are exactly proportional to the lengths of the planes.

272. If in a vertical semicircle any number of cords be drawn from any points whatever and all meeting in the lowest point of the semicircle, and a number of bodies be allowed to start along these cords at the same instant, they will all arrive at the bottom at the same instant, and at every instant of their descent they will all be in the circumference of a smaller circle.

Fig. 25.



Thus in the accompanying figure if ADP be a semicircle and BP, CP, DP, EP, FP, any cords, and balls be allowed to start simultaneously from A, B, C, D, E, and F, they will all arrive at P at the same instant. At the end of one-fourth the entire time they take to fall to P, A will have arrived at g, and the other bodies will be in the circumference gP; at the end of one-half the time of descent all will be in the circumference h, &c.

273. Bodies descending curves are subject to the same law as regards velocity as those on inclined planes, i. e., the terminal velocity is due only to the perpendicular fall.

274. The *Brachystochrone* (Greek *brachistos*, "shortest," and *chronos*, "time,") or *curve of quickest descent*, is a curve somewhat greater than a circular curve, being what mathematicians denominate a *cycloid*, or that which is described by a point in the circumference of a carriage wheel rolling along a plane.

275. Since Art. 270, the effect of gravity as an accelerating force on a body descending an inclined plane is to the effect of gravity on a body freely falling through the air as the height of the plane is to its length; we have accelerating force of gravity on inclined plane $: g :: h : l$; and hence

$$\text{accelerating force of gravity on inclined planes} = \frac{gh}{l},$$

where h = height of plane.
l = length.
g = effect of gravity = 32

Substituting this value of the effect of gravity in the formulas in Art. 26[illegible] we get the following formulas for the descent of bodies on inclined planes

FORMULAS FOR DESCENT OF BODIES ON INCLINED PLANES.

NO.	GIVEN.	TO FIND	FORMULAS.	CORRESPONDING FORMULA IN ART. 266.
1	g, h, l, t		$s = \frac{ght}{2l}$	I
2	g, h, l, v	s	$s = \frac{lv^2}{2gh}$	II
3	t, v		$s = \frac{1}{2}tv$	III
4	s, t		$v = \frac{2s}{t}$	VI
5	g, h, l, t	v	$v = \frac{ght}{l}$	IV
6	g, h, l, s		$v = \sqrt{\frac{2ghs}{l}}$	V
7	s, v		$t = \frac{2s}{v}$	VII
8	g, h, l, v	t	$t = \frac{lv}{gh}$	VIII
9	g, h, l, s		$v = \sqrt{\frac{2ls}{gh}}$	IX

276. When the body is projected down an inclined plane with a given initial velocity V; $s = Vt + \frac{ght^2}{2l}$ (10.) and $v = V + \frac{ght}{l}$ (11). When the body is projected up an inclined plane with a given initial velocity V; $s = Vt \frac{ght^2}{2l}$ (12.) and $v = V - \frac{ght}{l}$ (13.)

NOTE.—When a body is thrown up an inclined plane, the attraction of gravity acts as a uniformly retarding force as when a body is projected vertically into the air. In the case of the inclined plane the body will continue to rise with a constantly retarded motion until $Vt = \frac{ght^2}{2l}$ when it will remain stationary for an instant and then commence to descend. It will occupy the same time in coming down as in going up: its terminal velocity will be the same as its initial velocity, and it will have the same velocity at any given point of the plane both in ascending and descending.

EXAMPLE 338.—Through how many feet will a body fall in 15 seconds on an inclined plane which rises 7 feet in 40?

SOLUTION.

Here $t = 15$, $h = 7$, $l = 40$, and $g = 32$.

Then $s = \frac{ght^2}{2l} = \frac{32 \times 7 \times 15^2}{2 \times 40} = 630$ feet. *Ans.*

EXAMPLE 339.—Through how many feet must a body have fallen on an inclined plane, having a rise of 3 feet in 32, in order to acquire a terminal velocity of 1700 feet per second?

SOLUTION.

Here $g = 32$, $v = 1700$, $h = 3$, $l = 32$.

Then $s = \frac{lv^2}{2gh} = \frac{32 \times 1700^2}{2 \times 32 \times 3} = 481666\frac{2}{3}$ feet. *Ans.*

EXAMPLE 340.—What will be the velocity at the end of the 20th second, of a body falling down an inclined plane, having an inclination of 7 feet in 60 feet?

SOLUTION.

Here $g = 32$, $t = 20$, $h = 7$, and $l = 60$.

Then formula 5. $v = \frac{ght}{l} = \frac{32 \times 7 \times 20}{60} = 74\frac{2}{3}$ feet per second. *Ans.*

EXAMPLE 341.—On an inclined plane rising 3 ft. in 17, a body has fallen through one mile, what velocity has it then acquired?

SOLUTION.

Here $s = 1$ mile $= 5280$ ft. $h = 3$, $l = 17$ and $g = 32$.

Then formula VI. $v = \sqrt{\frac{2ghs}{l}} = \sqrt{\frac{2 \times 32 \times 3 \times 5280}{17}} = \sqrt{59632 \cdot 94} =$ $244 \cdot 17$ feet per second. *Ans.*

EXAMPLE 342.—In what time will a body falling down an inclined plane, having a rise of 7 feet in 16, acquire a terminal velocity of 777 feet per second?

SOLUTION.

Here $g = 32$, $h = 7$, $l = 16$, and $v = 777$.

Then formula 8. $t = \frac{lv}{hg} = \frac{16 \times 777}{32 \times 7} = 55\frac{1}{2}$ seconds. *Ans.*

EXAMPLE 343.—In what time will a body fall through 4780 feet on an inclined plane, having a rise of 3 feet in 4?

SOLUTION.

Here $g = 32$, $h = 3$, $l = 4$, and $s = 4780$.

Then formula 9. $t = \sqrt{\frac{2ls}{gh}} = \sqrt{\frac{2 \times 4 \times 4780}{32 \times 3}} = \sqrt{398 \cdot 3} = 19 \cdot 9$ seconds.

EXAMPLE 344.—If a body be projected down an inclined plane, having a rise of 8 feet in 15, with an initial velocity of 80 feet per second, through what space will it pass in 40 seconds?

SOLUTION.

Here $v = 80$, $g = 32$, $h = 8$, $l = 15$, and $t = 40$.

Then formula 10. $s = Vt + \frac{ght^2}{2l} = 40 \times 80 + \frac{32 \times 8 \times 40^2}{2 \times 15} = 3200 + 13653\frac{1}{3}$ $= 16853\frac{1}{3}$ ft. *Ans.*

EXAMPLE 345.—If a body be projected up an inclined plane having a rise of 5 feet in 16, with an initial velocity of 2000 ft. per second:—

1st. How far will it rise?
2nd. When will it again reach the bottom of the plane?
3rd. What will be its terminal velocity?
4th. Where will it be at the end of the 100th second?
5th. In what other moment of its flight will it have the same velocity as at the end of the 11th second of its ascent?

SOLUTION.

Here $h = 5$, $l = 16$, $g = 32$, and $v = 2000$.

Then formula 8, $t = \frac{lv}{gh} = \frac{16 = 2000}{5 \times 32} = 200$ seconds.

1st. Formula 12. $s = Vt - \frac{ght^2}{2l} = 200 \times 2000 - \frac{32 \times 5 \times 200^2}{2 \times 16} = 400000$ $- 200000 = 200000$ ft. *Ans.*

2nd. Ascent $= 200$ sec. + descent 200 sec. $= 400$ sec. *Ans.*

3rd. Terminal velocity = initial velocity = 2000 feet per sec. *Ans.*

4th. Formula 12. $s = Vt - \frac{ght^2}{2l} = 100 \times 20000 - \frac{32 \times 5 \times 100^2}{2 \times 16} - 200000$ $- 50000 = 150000$ = elevation at end of 100th sec. *Ans.*

5th. $400 - 11 = 389$th second.

EXERCISE.

346. On an inclined plane rising 5 feet in 19, through what space will a body descend in half a minute? *Ans.* $3789\frac{9}{19}$ ft.

347. On an inclined plane rising 3 feet in 13, what velocity will a descending body acquire in 39 seconds? *Ans.* 288 feet per second.

348. What time does a body require to descend through 3800 feet on a plane rising 19 feet in 32? *Ans.* 20 seconds.

349. If a body be projected down an inclined plane, having a fall of 7 in 11 with an initial velocity of 50 feet per second, what will be its velocity at the end of the 44th second? *Ans.* 946 feet per second.

350. If a body be thrown down an inclined plane having a rise of 13 feet in 32 with an initial velocity of 100 feet per second, through how many feet will it descend in 130 seconds? *Ans.* 122850 feet.

351. If a body be projected up an inclined plane, having a fall of 5 feet in 8, with an initial velocity of 800 feet per second :—

1st. How far will it rise?

2nd. In how many seconds will it again reach the bottom of the plane?

3rd. What will be its terminal velocity?

4th. Where will it be at the end of the 68th second?

5th. In what other moment of its flight will it have the same velocity as at the end of the 37th second of its ascent?

Ans. 1st. Rise = 16000 ft.; 2nd. Time of flight = 80 seconds; 3rd. Terminal velocity = 800 feet per second; 4th Elevation at end of 68th sec. = 8160 feet; 5th. At the end of the 43d second.

352. A body rolls down an inclined plane, being a rise of 7 ft. in 20—when it has descended through f feet, another body commences to descend at a point m feet beneath it. Through how many feet will the second body descend before the first body passes it?

Ans. $\frac{m^2}{4f}$

PROJECTILES.

277. A projectile is a solid body to which a motion has been communicated near the surface of the earth, by any force, as muscular exertion, the action of a spring, the explosive effects of gunpowder, &c., which ceases to act the moment the impulse has been given.

278. A projectile is at once acted upon by two forces,—

1st. The projectile force which tends to make the body move over equal spaces in equal times; and

2nd. The force of gravity, which tends to make the body move towards the centre of the earth over spaces which are proportional to the squares of the times.

Under the joint influences of these two forces the projectile describes a curve, which *in theory* is the parabola, but which *in practice* departs very materially from that figure.

NOTE 1.—The *parabola* is that curve which is produced by cutting a cone paralled to its side.

NOTE 2.—The parabolic theory is based upon three suppositions, all of which are more or less inaccurate.

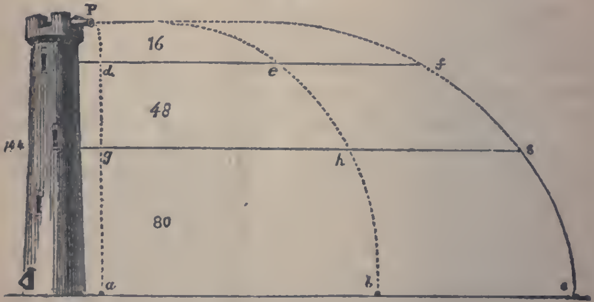
1st. That the force of gravity is the same in every part of the curve described by the projectile.

2nd. That the force of gravity acts in parallel lines.
3rd. That the projectile moves through a non-resisting medium.
The first and second of these suppositions differ so insensibly from truth that they may be assumed to be absolutely correct, but the resistance of the atmosphere so materially affects the motions of all bodies, especially when their velocities are considerable, that it renders the parabolic theory practically useless.

279. When a body is projected horizontally forward, the horizontal motion does not interfere with the action of gravity, — the projectile descending with the same rapidity while moving forward, that it would if acted upon by gravity alone.

NOTE.—The accompanying figure represents a tower 144 feet in height. Now if three balls *a*, *b*, and *c*, be made to start simultaneously from P, one dropping vertically, one being projected forward with sufficient force to carry it horizontally half a mile, and the third with sufficient force to carry it horizontally to any other distance, say one mile, all three balls will reach the ground, provided it be a horizontal plane, at the same instant. Thus each ball will have fallen 16 feet at the end of the 1st second, and they will simultaneously cross the line *def*. At the end of the 2nd second they have each descended 64 feet, and are respectively at *g*, *h*, and *s*, &c.

Fig. 26.



280. According to the parabolic theory :—

1st. The projectile rises to the greatest height, and remains longest before it again reaches the ground, when thrown vertically upwards.

2nd. The distance or range over a horizontal plane is greatest, when the angle of elevation is 45°.

3rd. With an initial velocity of 2000 per second, the projectile should go about 24 miles.

NOTE.—The first of these laws is found by experiment to be absolutely correct, and the second is not far from the truth, the greatest range taking place at an angle of elevation somewhat less than 45°.

The difference between the third law and the result of experiment is prodigious; for no projectile, however great its initial velocity may have been, has ever been thrown from the surface of the earth to a horizontal distance of 5 miles.

281. Whatever may be the initial velocity of projection, it is speedily reduced by atmospheric pressure to a velocity not exceeding 1280 feet per second.

NOTE 1.—This arises from the fact that atmospheric air flows into a vacuum with a velocity of only 1280 feet per second, so that when a ball moves with a greater velocity than this, it leaves a vacuum behind it into which the strongly compressed air in front tends powerfully to force it.

NOTE 2.—From experiments made with great care, is has been ascertained that when the velocity of a ball or other projectile is 2000 feet per second, the ball meets with an atmospheric resistance equal to 100 times its own weight.

NOTE 3.—Another great irregularity in the firing of balls arises from the fact that the ball deviates more or less to the right or left, sometimes crossing the direct line several times in a very short course. This deflection sometimes amounts from $\frac{1}{6}$ to $\frac{1}{4}$ of the whole range, or as much as 300 or 400 yards in a mile when there is considerable windage; i. e., when the ball is too small for the calibre of the gun.

282. The motion of projectiles has recently been investigated with much care, with the view of deducing a new theory in which the resistance of the air should be taken into account. The following are the most important results :—

WHEN THE BODY IS THROWN VERTICALLY UPWARDS INTO THE AIR.

I. *The time of ascent is less than the time of descent.*

II. *The velocity of descent is less than that of ascent.*

III. *The terminal velocity is less than the initial velocity.*

IV. *The velocity of descent is not infinitely accelerated since when the velocity becomes very great, the resistance of the atmosphere becomes so great as to counterbalance the accelerating force of gravity, and the velocity of the descending body is thenceforth uniform.*

WHEN THE PROJECTILE IS THROWN AT AN ANGLE OF ELEVATION.

I. *The ascending branch of the curve is longer than the descending branch.*

II. *The time of describing the ascending branch is less than that of describing the descending branch.*

III. *The descending velocity is less than the ascending.*

IV. *The terminal velocity is less than the initial.*

V. *The direction of the descending branch is constantly approximating to a vertical line, which it never reaches.*

VI. *The descending velocity is not infinitely accelerated, but, as in case of a body falling vertically, becomes constant after reaching a certain limit.*

VII. *The limit of the velocity of descent is different in different bodies, being greatest when they are dense, and increasing with the diameter of spherical bodies.*

283. The explosive force of gunpowder, fired in a piece of ordnance, is equal to 2000 atmospheres, or 30000 lbs. to the square inch, and it tends to expand itself with a velocity of 5000 feet per second.

NOTE.—Gunpowder is an intimate mixture of 6 parts saltpetre, 1 part charcoal, and 1 part sulphur. In firing good perfectly dry gunpowder, the ignition takes place in a space of time so short as to appear instantaneous. 1 cubic inch of powder produces 800 cubic inches of cold gas, and, as at the moment of explosion the gas is red hot, we may safely reckon the expansion as about 1 into 2000.

284. The greatest initial velocity that can be given to a cannon ball is little more than 2000 feet per second, and that only at the moment it leaves the gun.

Note.—The velocity is greatest in the longest pieces; thus Hutton found the velocity of a ball of given weight, fired with a given charge of powder to be in proportion to the fifth root of the length of the piece.

285. The velocities communicated to balls of equal weights, from the same piece of ordnance, by unequal weights of powder, are as the square roots of the quantities of powder.

286. The velocities communicated to balls of different weights and of the same dimensions, by equal quantities of powder, are inversely proportional to the square roots of the weights of the balls.

287. The depth to which a ball penetrates into an obstacle is in proportion to the density and diameter of the ball and the square root of the velocity with which it enters.

NOTE 1.—An 18-pound ball with a velocity of 1200 feet per second penetrates 34 inches into dry oak, and a 24-pound ball with a velocity of 1300ft. per second penetrates 13 feet into dry earth.

NOTE 2.—The length of guns has been much reduced in all possible cases. Field pieces are now seldom made of greater length than 12 or 14 calibres (diameter of the ball). The maximum charge of powder has also been diminished very greatly—now seldom exceeding one-third, and often being as low as one-twelfth of the weight of the ball.

288. The following rule, obtained from experiment, has been given, to find the velocity of any shot or shell, when the weight of the charge of powder and also that of the shot are known.

RULE.

Divide three times the weight of powder by the weight of the shot, multiply the square root of the quotient by 1600, and the product will be the velocity per second in feet.

Or if p = charge of powder in lbs., w = weight of ball in lbs. and v = velocity per second in feet; then $v = 1600 \times \sqrt{\left(\frac{3p}{w}\right)}$

Example 353.—What is the velocity of a ball weighing 48 lbs., fired by a charge of 4 lbs. of powder?

SOLUTION.

Here $p = 4$ and $w = 48$.

Then $v = 1600 \times \sqrt{\left(\frac{3p}{w}\right)} = 1600 \times \sqrt{\left(\frac{3 \times 4}{48}\right)} = 1600 \times \sqrt{\left(\frac{1}{4}\right)}$

$= 1600 \times \frac{1}{2} = 800$ feet per second. *Ans.*

Example 354.—With what velocity will a charge of 7 lbs. of powder throw a ball weighing 32 lbs.?

SOLUTION.

Here $p = 7$ and $w = 32$.

Then $v = 1600 \times \sqrt{\frac{3p}{w}} = 1600 \times \sqrt{\frac{3 \times 7}{32}} = 1600 \times \sqrt{\cdot 65625} = 1600$

$\times \cdot 81 = 1296$ feet per second. *Ans.*

Example 355.—If 4 lbs. of powder throw a ball of 16 lbs. in weight with a velocity of 1200 ft. per second, what amount of powder would throw the same ball with a velocity of 600 ft. per second?

SOLUTION.

Art. 285. vel. : vel. : : $\sqrt{}$ (weight of powder) : $\sqrt{}$ (weight of powder); or $1200 : 600 : : \sqrt{4} : \sqrt{x}$, and hence $x = 1$ lb. *Ans.*

Example 356.—If 3 lbs. of powder throw a ball 6 inches in diameter and weighing 32 lbs., with a velocity of 850 feet per second, with what velocity will the same charge throw another ball of the same dimensions but weighing only 9 lbs.?

SOLUTION.

Art. 286. $\sqrt{9} : \sqrt{32} : : 850 : x$, or $3 : 5 \cdot 65 : : 850 : x$.
And hence $x = 1600$ feet. *Ans.*

EXERCISE.

357. With what velocity will a charge of 11 lbs of powder throw a cannon ball weighing 24 lbs.?

Ans. 1876 feet per second

358. With what velocity will a charge of 9 lbs. powder throw a ball weighing 36 lbs.? *Ans.* 1385 feet per second.

359. If 7 pounds of powder throw a ball with a velocity of 1000 feet per second, what charge will throw the same ball with a velocity of 1500 feet per second? *Ans.* 15¾ lbs.

360. If a certain charge of powder throw a 10-inch ball weighing 20 lbs. with a velocity of 973 feet per second, with what velocity will the same charge throw a ball of the same dimensions weighing only 25 lbs.?

Ans. 870 feet per second.

CIRCULAR MOTION.

289. Centrifugal force (Lat. *centrum*, "the centre," and *fugio.* "I flee"), is that force by which a body moving in a circle tends to fly off from the centre.

NOTE.—Since a body moving in a circle would, if not restrained by other forces, fly off in a tangent to that circle, centrifugal force is sometimes called *tangential* force.

290. Centripetal force (Lat. *centrum*, "the centre," and *peto*, "I seek or rush to"), is that force by which a body moving in a circle is held or attracted to the centre.

291. When a body is at once acted upon by both centrifugal and centripetal force, it moves in a curve, and the form of this curve depends upon the relative intensities of the two forces: i. e., if the two be equal at all points, the curve will be a circle, and the velocity of the body will be uniform; but if the centrifugal force, at different points of the body's orbit, be inversely as the square of the distance from the centre of gravity, the curve will be an ellipse, and the velocity of the body will be variable.

292. When a body rotates upon an axis, all its parts revolve in equal times; hence the velocity of each particle increases with its perpendicular distance from the axis, and so also does its centrifugal force.

NOTE 1.—As long as the centrifugal force is less than the cohesive force by which the particles are held together, the body can preserve itself; but, as soon as the centrifugal force exceeds the cohesive, the parts of the rotating mass fly off in directions which are tangents to the circles in which they were moving.

NOTE 2.—We have examples of the effects of centrifugal force in the destructive violence with which rapidly revolving grindstones burst and fly to pieces, the expulsion of water from a rotating mop, the projection of a stone from a sling, the action of the conical pendulum or governor in regulating the supply of steam in an engine, &c, &c.

293. When the velocity and radius are constant, the centrifugal force is proportional to the weight.

294. When the radius is constant, the centrifugal force varies as the square of the velocity.

NOTE.—At the equator the centrifugal force of a particle is $\frac{1}{289}$ of its gravity or weight, and from the equator it diminishes as we approach the poles where it becomes 0. It follows that if the earth were to revolve 17 times faster than it does, the centrifugal force at the equator would be equal to gravity, and a body would not fall there at all. If the earth revolved still more rapidly, the water, inhabitants, &c., would be whirled away into space, and the equatorial regions would constitute an impassable zone of sterility.

295. When the velocity is constant, the centrifugal force is inversely proportional to the radius.

296. When the number of revolutions is constant, the centrifugal force is directly proportional to the radius.

297. Let $c =$ centrifugal force, $v =$ the velocity per second in feet, $r =$ radius in feet, $g = 32$, $w =$ weight, and $n =$ the number of revolutions per second.

Then $c = \frac{wv^2}{gr}$ (I), $r = \frac{wv^2}{cg}$ (II), $w = \frac{cgr}{v^2}$ (III), $v = \sqrt{\left(\frac{cgr}{w}\right)}$ (IV).

Also, since $v = r \times 2 \times 3{\cdot}1416 \times n$, $v^2 = r^2 \times 4 \times (3{\cdot}1416)^2 \times n^2$, and hence formula I.: $c = \frac{w \times r^2 \times 4 \times (3{\cdot}1416)^2 \times n^2}{gr}$, and reducing this we get $c = wrn^2 \times 1{\cdot}2345$ (V), $w = \frac{c}{rn^2 \times 1{\cdot}2345}$ (VI), $r = \frac{wn^2 \times 1{\cdot}2345}{c}$ (VII), and $n = \sqrt{\left(\frac{c}{wr \times 1{\cdot}2345}\right)}$ (VIII).

EXAMPLE 361.—What is the centrifugal force exerted by a body weighing 10 lbs. revolving with a velocity of 20 feet per second in a circle 8 feet in diameter?

SOLUTION.

Here $w = 10$, $v = 20$, $r = 4$, and $g = 32$.

Then $c = \frac{wv^2}{gr} = \frac{10 \times 20^2}{32 \times 4} = \frac{10 \times 400}{32 \times 4} = 31\frac{1}{4}$ lbs. *Ans.*

EXAMPLE 362.—What centrifugal force is exerted by a body weighing 15 lbs. revolving in a circle 3 feet in diameter and making 100 revolutions per minute?

SOLUTION

Here $w = 15$, $r = 1{\cdot}5$, $n = \frac{100}{60} = 1\frac{2}{3}$

Then formula V.: $c = wrn^2 \times 1{\cdot}2345 = 15 \times 1{\cdot}5 \times (1\frac{2}{3})^2 \times 1{\cdot}2345 = 77{\cdot}15625$ lbs. *Ans.*

EXAMPLE 363.—A body weighing 40 lbs. revolves in a circle 4 feet in diameter; in order that its centrifugal force may be 1847 lbs., what must be its velocity and number of revolutions per second?

SOLUTION

Here $w = 40$ lbs $r = 2$, and $c = 1847$.

Then formula VIII.: $n = \sqrt{\left(\frac{c}{wr \times 1{\cdot}2345}\right)} = \sqrt{\left(\frac{1847}{40 \times 2 \times 1{\cdot}2345}\right)}$
$= \sqrt{18{\cdot}7019} = 4{\cdot}32$ = number of revolutions per second, and hence revolutions per minute $= 256{\cdot}8$.

Also $v = 4 \times 3{\cdot}1416 \times 4{\cdot}32 = 54{\cdot}28$ feet per second.

EXAMPLE 364.—The diameter of a grindstone is 4 feet, its weight half a ton, and the centrifugal force required to burst it is 45 tons: with what velocity must it revolve, and how many revolutions must it make per minute in order to burst?

SOLUTION.

Here $w = \frac{1}{2}$, $c = 45$, and $r = 2$.

Then formula VIII.: $n = \sqrt{\left(\frac{c}{wr \times 1{\cdot}2245}\right)} = \sqrt{\left(\frac{45}{\frac{1}{2} \times 2 \times 1{\cdot}2345}\right)}$
$= \sqrt{36{\cdot}452} = 6{\cdot}03$ = revolutions per second, and hence $6{\cdot}03 \times 60 = 361{\cdot}8$ = the revolutions per minute.

Also velocity $= 4 \times 3{\cdot}1416 \times 6{\cdot}03 = 75{\cdot}775$ feet per second.

EXERCISE.

365. If a ball weighing 4 lbs. be attached to a string $2\frac{1}{2}$ feet long and whirled round in a circle so as to make 120 revolutions per minute,—what must be the strength of the string in order to just keep the ball from flying off? *Ans.* 49·38 lbs.

366. A ball weighing 2 lbs. is attached to a string $3\frac{1}{2}$ feet long and capable of resisting a strain of 200 lbs.; if the ball be whirled in a circle with the whole length of the string as radius, how many revolutions per minute must it make in order to break the string? *Ans.* $288\frac{2}{3}$ revolutions.

367. A ball is whirled in a circle, with a velocity of 64 feet per second, by means of a string 4 feet in length and capable of resisting a strain of 840 lbs.; what must be the weight of the ball in order to break the string? *Ans.* $26\frac{1}{4}$ lbs.

368. What is the centrifugal force exerted by a body weighing 20 lbs. revolving in a circle 10 feet in diameter and making 2·8 revolutions per second? *Ans.* 967·848 lbs.

369. What is the centrifugal force exerted by a body weighing 8 lbs. and revolving in a circle 20 feet in diameter with a velocity of 100 feet per second? *Ans.* 250 lbs.

ACCUMULATED WORK.

298. Work is required to set a body in motion or to bring a moving body to a state of rest. For example, when a common engine is first set in action a considerable portion of the work of the engine goes to give motion to the fly-wheel and other parts of the machinery; and before the engine can come to a state of rest, all of this accumulated work must be destroyed by friction, atmospheric resistance, &c.

299. To find the work accumulated in a moving body:—

RULE.

I. Find the height in feet from which the body must have fallen to have acquired the given velocity.

II. Multiply the number thus found by the weight of the body in pounds.

Or let U = units of work accumulated, v = velocity, w = the weight in lbs., and $g = 32$.

Then Art. 266, *since* $s = h = \frac{v^2}{2g}$

$$U = hw = \frac{v^2}{2g} \times w = \frac{v^2 w}{2g}$$

EXAMPLE 370.—A ball weighing 10 lbs. is projected on smooth ice with a velocity of 100 feet per second: assuming the friction to be $\frac{1}{15}$ of the weight of the ball, and neglecting atmospheric resistance; over what space will it pass before coming to a state of rest?

SOLUTION.

Here $v = 100$, $w = 10$, and $g = 32$.

Then $U = \frac{v^2 w}{2g} = \frac{100^2 \times 10}{2 \times 32} = \frac{100000}{64} = 1562\frac{1}{2}$ = units of work accumulated in the ball.

Also $\frac{1}{15} \times 10 \times 1 = \frac{2}{3}$ = units of work destroyed by friction in moving the ball through 1 foot.

Therefore the number of feet $= 1562\frac{1}{2} \div \frac{2}{3} = 2343\frac{3}{4}$. *Ans.*

EXAMPLE 371.—A train weighs 100 tons and has a velocity of 40 miles per hour when the steam is turned off: how far will it ascend a plane having an inclination of ½ in 100, taking friction as 11 lbs. per ton, and neglecting the resistance of the atmosphere?

SOLUTION.

Here $v = 40$ miles per hour $= \frac{40 \times 5280}{60 \times 60} = 58\frac{2}{3}$ feet per second, $w = 100$ tons $= 200000$ lbs. and $g = 32$.

Then $U = \frac{v^2 w}{2g} = \frac{(58\frac{2}{3})^2 \times 200000}{2 \times 32} = \frac{3441\frac{7}{9} \times 200000}{64} = 3441\frac{7}{9} \times 3125 =$ $10755555\frac{5}{9}$ = units of work accumulated in the train.

Work of friction $= 100 \times 11 = 1100$ units to each foot.

Work of gravity $= \frac{1}{200} \times 200000 = 1000$ units to each foot.

Work destroyed by resistances, i.e., friction and gravity, in moving the train over one foot $= 1100 + 1000 = 2100$ units.

Therefore number of feet $= \frac{10755555\frac{5}{9}}{2100} = 5121{\cdot}69$ feet = nearly one mile.

EXAMPLE 372.—If a car weighing 3 tons, and moving at the rate of 10 feet per second on a level rail, pass over 500 feet before it comes to a state of rest, what is the resistance of friction per ton?

SOLUTION.

Work accumulated in car $= \frac{10^2 \times 6000}{2 \times 32} = \frac{600000}{64} = 9375$ units.

Work of friction = friction $\times$ 500.

Therefore friction $\times 500 = 9375$, and hence friction $= \frac{9375}{500} = 18\frac{3}{4}$ lbs. on whole car.

Then friction per ton $= 18\frac{3}{4} \div 3 = 6\frac{1}{4}$ lbs. *Ans.*

EXERCISE.

373. A train weighing 90 tons is moving at the rate of 30 miles per hour when the steam is shut off: how far will it go before stopping, on a level plane, assuming the coefficient of friction to be $\frac{1}{200}$? *Ans.* 6050 feet, or $1\frac{7}{48}$ miles.

374. A train weighing 80 tons has a velocity of 30 miles per hour when the steam is turned off: how far will it ascend a plane rising 7 feet in 1000—taking friction, as usual, and neglecting atmospheric resistance?

Ans. 2880·95 feet.

375. Required the units of work accumulated in a body whose weight is 29 lbs. and velocity 144 feet per second?

Ans. 9396.

376. A ball weighing 15 lbs. is projected on a level plane, with a velocity of 90 feet per second : assuming friction to be equal to $\frac{1}{10}$ of the weight of the ball, how far will it go before it comes to a state of rest? *Ans.* 1265·625 feet.

377. A train weighing 90 tons has a velocity of 100 feet per second when the steam is turned off: how far will it go on a level plane, assuming friction to be equal to 12 lbs. per ton, and neglecting atmospheric resistance?
Ans. 26041$\frac{2}{3}$ feet.

378. A ball weighing 20 lbs. is thrown along a perfectly smooth plane of ice with a velocity of 60 feet per second : how far will it go before stopping if the friction be $\frac{1}{20}$ of the weight?
Ans. 1125 feet.

379. A train weighing 100 tons has a velocity of 25 feet per second when the steam is turned off: how far will it descend an incline of 3 in 100, taking friction to be equal to 12 lbs. per ton? *Ans.* 3255·2 feet.

380. Required the work accumulated in a body which weighs 50 lbs. and which is moving with a velocity of 70 feet per second. *Ans.* 3828$\frac{1}{8}$ units.

381. What work is accumulated in a ram weighing 2000 lbs. falling with a velocity of 40 feet per second?
Ans. 50000 units.

THE PENDULUM.

300. A pendulum consists of a heavy body suspended by a thread or slender wire, and made to vibrate in a vertical plane.

301. When the body is regarded as a point, and the thread or wire without weight, the pendulum is called a *Simple Pendulum.*

302. A *Compound Pendulum* or *Material Pendulum* consists of a heavy body suspended by a ponderable wire or thread.

303. The motion of the pendulum from one extremity to the other of the arc in which it moves, is called an *oscillation* or a *vibration.*

304. The *amplitude* of the arc of vibration is measured by the number of degrees, minutes and seconds through which the pendulum oscillates.

305. The *duration* of a vibration is the space of time occupied by the pendulum in swinging from one extremity to the other of the arc of vibration.

306. The length of the pendulum is the distance between the centre of suspension and the centre of oscillation.

307. The *centre of suspension* is the point round which the pendulum moves as a centre.

308. The *centre of oscillation* is that point in a vibrating body, into which, if all the matter were collected, the time of vibration would remain unchanged.

NOTE 1.—If a bar of iron or any other substance be suspended by one extremity and made to vibrate, it constitutes a compound pendulum. Now, if the several particles composing the rod were free to move separately, those nearer the centre of suspension would vibrate more rapidly than those more remote; but since the pendulum is a solid body, all of its particles must vibrate in the same time, and hence the motion of those molecules which are nearer the centre of suspension is retarded, while that of the more remote parts is accelerated. Somewhere in the rod, however, there must be a point or particle so situated with respect to the centre of suspension, and the other parts of the rod, that the accelerating effect of the particles above it is exactly neutralized by the retarding force of the molecules below it; and, consequently, this particle or point vibrates in exactly the same time that it would occupy if liberated from all connection with the parts above, below and around it, and were set swinging by an imponderable thread—this point is called the centre of oscillation.

NOTE 2.—The centre of oscillation in a vibrating mass coincides with what is called the *centre of percussion*. The centre of percussion is that point in a revolving body, which, upon striking against an immovable obstacle, will cause the whole of the motion accumulated in the revolving body to be destroyed, so that, at the moment of impact, the body would have no tendency to move in any direction. In a rod of inappreciable thickness the centre of percussion is two-thirds of the length of the rod from the axis about which it moves.

309. The centres of suspension and oscillation in the pendulum are interchangeable, i. e., if the pendulum be inverted and suspended by its centre of oscillation, the former point of suspension will become the centre of oscillation, and the pendulum will vibrate in precisely the same time.

LAWS OF THE OSCILLATION OF THE PENDULUM.

310. The duration of an oscillation is independent of its amplitude, provided it does not exceed 4° or 5°.

NOTE 1.—This fact is commonly stated by saying that the vibrations of the pendulum are *isochronous;* i. e., equal-timed. Thus a pendulum of a given length will oscillate through an arc of 5° in the same time it would have required to vibrate through an arc of 0.1° although the amplitude of the vibration is in the one case 50 times as great as in the other. This arises from the fact that the pendulum in moving through the larger arc falls through a greater vertical distance, and hence acquires a greater velocity.

NOTE 2.—Strictly speaking, the oscillations of the pendulum are isochronous only when the curve in which they move is a cycloid. When, however, the common pendulum vibrates in very small arcs, as of 2° or 3°, the oscillations are, for all practical purposes, isochronous.

311. The duration of the vibration is independent of the weight of the ball and the nature of its substance.

312. Two pendulums of equal lengths perform an equal number of vibrations in the same period of time.

313. Two pendulums of unequal lengths perform an unequal number of vibrations in the same period of time —the longest pendulum performing the smallest number of oscillations.

314. Pendulums of unequal lengths vibrate in times which are to one another as the square roots of their lengths.

315. A *seconds pendulum* is one that performs exactly sixty vibrations in a minute, or one vibration in one second.

316. The time occupied by a vibration depends :—

1st. Upon the length of the pendulum ; and

2nd. Upon the intensity of the force of gravity.

NOTE.—Since the earth is not an exact sphere, being flattened at the poles, the surface of the earth at the poles is nearer to the centre than at the equator. Hence the intensity of the force of gravity is less at the equator than at the poles, and a pendulum that beats seconds at the equator must be lengthened in order to beat seconds as it is carried towards the poles. In point of fact, a seconds pendulum at the poles is about one-fifth of an inch longer than a seconds pendulum at the equator. The following Table shows the length of the seconds pendulum at different parts of the earth's surface, and also the magnitude of the force of gravity; i.e., the velocity which the force of gravity will impart to a dense body in falling for one entire second.

Place.	Latitude.	Length of Seconds Pendulum.	Velocity acquired by a body falling one second.
St Thomas	0° 24′	39·01 inches	3849·86 inches
Ascension	7° 55′	39·02 "	3842·86 "
New York	40° 42′	39·10 "	385·978 "
Paris	48° 50′	39·12 "	386·076 "
London	51° 31′	39·13 "	386·174 "
Spitzbergen	79° 50′	39·21 "	386·984 "

NOTE.—In Canada the seconds pendulum is about 39·11 in. in length.

317. The pendulum is applied to three purposes:—

1st. As a measure of time;

2nd. As a measure of the force of gravity: and

3rd. As a standard of measure.

NOTE.—The pendulum is used as a measure of time by attaching it to clock-work, which serves the double purpose of registering its oscillations and restoring to the pendulum the motion lost in its vibration by friction and atmospheric resistance. The use of the pendulum as a standard of measure will be seen from the following statements, viz.:

1st. A *pound pressure* means that amount of pressure which is exerted towards the earth, in the latitude of London and at the level of the sea, by the quantity of matter called a *pound*.

2nd. A *pound of matter* means a quantity equal to that quantity of pure water which, at the temperature of 62 deg. Fahrenheit, would occupy 27·727 cubic inches.

3rd. A *cubic inch* is that cube whose side, taken 39·1393 times, would measure the effective length of a London seconds pendulum.

4th. A *seconds* pendulum is that which, by the unassisted and unopposed effect of its own gravity, would make 86400 vibrations in an artificial solar day, or 86163·09 in a natural sidereal day.

318. *If t = the time of oscillation, l = the length of the pendulum, g = the force of gravity; i. e., the velocity which the force of gravity would impart to a dense body falling through one entire second, and π = 3·1416; i. e., the ratio between the diameter of a a circle and its circumference.*

Then $t = \pi\sqrt{\left(\frac{l}{g}\right)}$ (I.) $l = \frac{t^2 g}{\pi^2}$ (II.) *and* $g = \frac{l\pi^2}{t^2}$ (III.)

When t = one second, formulas (II.) *and* (III.) *respectively become*

$l = \frac{g}{\pi^2}$ (IV.) *and* $g = l\pi^2$ (V.)

319. To find the time in which a pendulum of given length will vibrate, or the length of a pendulum that vibrates in a given time:—

Let l = the length and t = the time. Then since (*Art.* 314) *the times are as the square roots of the lengths, and in Canada the seconds pendulum is* 39·11 *inches in length—we have*

$t : 1 :: \sqrt{l} : \sqrt{(39{\cdot}11)}$; *and hence*

$t = \sqrt{\left(\frac{l}{39{\cdot}11}\right)}$ (VI.), *and* $l = t^2 \times 39{\cdot}11$. (VII.)

320. To find the number of vibrations which a pendulum of given length will lose by decreasing the force of gravity,

i. e., by carrying the pendulum to the top of a mountain or other elevation.

Let n = the number of vibrations performed at the earth's surface in the given time, n' = the number of vibrations lost in the same time, r = the radius of the earth, = 4000 miles, and h = the height of the mountain in miles or fraction of a mile; then

$n' = \frac{nh}{r}$ (VIII), *and hence* $h = \frac{n'r}{n}$ (IX.)

321. To find the number of vibrations which a pendulum of given length will gain in a given time by shortening the pendulum.

Let l = the given length of the pendulum, and l' = the decrease in length; also let n = the number of vibrations performed in the given time, and n' = the number of vibrations gained in the same time; then

$$n' = \frac{n'l}{2l} \text{ (x.) and } l' = \frac{2\,n'l}{n} \text{ (xi.)}$$

EXAMPLE 382.—How many vibrations will a pendulum 36 inches long made in one minute?

SOLUTION.

Formula VI: $t = \sqrt{\left(\frac{l}{39 \cdot 11}\right)} = \sqrt{\left(\frac{36}{39 \cdot 11}\right)} = \sqrt{\cdot 9204} = \cdot 959$ seconds.

Hence the number of vibrations $= 60 \div \cdot 959 = 62 \cdot 56$.

EXAMPLE 383.—Required the length of a pendulum that makes 80 vibrations in a minute.

SOLUTION.

Here $t = \frac{60}{80} = \frac{3}{4}$

Then formula VII. $l = t^2 \times 39 \cdot 11 = (\frac{3}{4})^2 \times 39 \cdot 11 = \frac{9}{16} \times 39 \cdot 11 =$

$21 \cdot 999$ inches.

EXAMPLE 384. In what time will a pendulum 60 inches long vibrate?

SOLUTION

Formula VI.; $t = \sqrt{\left(\frac{1}{39 \cdot 11}\right)} = \sqrt{\left(\frac{60}{39 \cdot 11}\right)} = \sqrt{1 \cdot 5341} = 1 \cdot 239$ seconds.

Ans.

EXAMPLE. 385.—A pendulum which beats seconds is taken to the top of a mountain one mile high: how many seconds will it lose in 6 hours?

SOLUTION.

Here $n = 6 \times 60 \times 60$, $h = 1$, and $r = 4000$.

Then formula (VIII.): $n' = \frac{nh}{r} = \frac{6 \times 60 \times 60 \times 1}{4000} = \frac{21600}{4000} = 5 \cdot 4$. *Ans.*

EXAMPLE 386.—If a clock lose 1 minute in 24 hours, how much must the pendulum be shortened to make it keep true time?

SOLUTION.

Here $n = 24 \times 60 \times 60$, $n' = 60$, and $l = 39 \cdot 11$.

Then formula XI.: $l' = \frac{2\ n'l}{n} = \frac{2 \times 60 \times 39 \cdot 11}{24 \times 60 \times 60} = 0 \cdot 0543$ or about $\frac{1}{18}$th of an inch. *Ans.*

EXAMPLE 387.—Through what distance will a heavy body fall in Canada during one entire second, and what will be its terminal velocity?

SOLUTION.

Here $t = 1$, and $l = 39 \cdot 11$.

Then formula V. $g = l\pi^2 = 39 \cdot 11 \times (3 \cdot 1416)^2 = 39 \cdot 11 \times 9 \cdot 86965056 = 386 \cdot 002$ inches = terminal velocity.

Hence the space passed through $= \frac{0 + 386 \cdot 002}{2} = 193 \cdot 001$ inches = $16 \cdot 0835$ feet. *Ans.*

EXAMPLE 388.—What must be the length of a pendulum in order vibrate ten times in a minute?

SOLUTION,

Here $t = \frac{60}{10} = 6$ seconds.

Then formula VII. $l = t^2 \times 39 \cdot 11 = 10^2 \times 39 \cdot 11 = 100 \times 39 \cdot 11 = 3911$ in. $= 326$ feet nearly. *Ans.*

EXAMPLE 389.—A pendulum which vibrates seconds at the surface of the earth is taken to the top of the mountain and is there found to lose 18 seconds in a day of 24 hours: required the height of the mountain.

SOLUTION.

Here $n' = 24 = 24 \times 60 \times 60'$ $n = 18$, and $r = 4000$.

Then $h = \frac{n'r}{n} = \frac{18 \times 4000}{24 \times 60 \times 60} = \frac{5}{6}$ miles $= 4400$ feet. *Ans.*

EXAMPLE 390. If a seconds pendulum be shortened $1\frac{1}{4}$ inch, how many vibrations will it make in one minute?

SOLUTION.

Here $n = 60$, $l = 39{\cdot}11$, and $l' = 1{\cdot}25$.

Then formula X.; $n' = \frac{nl'}{2l} = \frac{60 \times 1{\cdot}25}{2 \times 39{\cdot}11} = 0{\cdot}958 =$ the number of vibrations gained; hence the number of vibrations made $= 60{\cdot}958$. *Ans.*

EXAMPLE 391.—What will be the velocity acquired by a heavy body falling during one entire second in the latitude of Spitzbergen?

SOLUTION.

Here $t = 1$, and by the table Art. 316, $l = 39{\cdot}21$.
Then $g = l\pi^2 = 39{\cdot}21 \times (3{\cdot}1416^2 = 39{\cdot}21 \times 9{\cdot}86965 = 386{\cdot}988$ inches. *Ans.*

EXERCISE.

392. What must be the length of a pendulum in the latitude of Canada in order that it shall vibrate once in 3 seconds? *Ans.* 351·997 inches.

393. A pendulum that vibrates seconds at the surface of the earth is carried to the summit of a mountain 3 miles in height: how many seconds will it lose in 24 hours? *Ans.* 64·8.

394. In what time will a pendulum 10 inches in length vibrate? *Ans.* ·505 seconds.

395. What velocity will a heavy body falling in the latitude of New York acquire in one entire second? *Ans.* 385·903.

396. If a clock lose 10 minutes in 24 hours, how much must the pendulum be shortened in order that it shall keep correct time? *Ans.* ·543 or over ½ of an inch.

397. If a seconds pendulum be shortened 5 inches, how many vibrations will it make in a minute? *Ans.* 63·83,

398. A pendulum which vibrates seconds at the surface of the earth is carried to the summit of a mountain, where it is observed to lose 30 seconds in 24 hours: required the height of the mountain. *Ans.* 7333·3 feet.

399. In what time will a pendulum 100 inches long vibrate? *Ans.* 1·59 seconds.

400. Required the length of a pendulum which makes 120 vibrations per minute? *Ans* 9·77 inches.

401. Through how many feet will a body fall in one second, and what will be its terminal velocity at the end of that portion of time in the latitude of Paris?
Ans. Terminal velocity = 386·1 in.
Space passed over = 16·0875 ft.

CHAPTER VIII.

HYDRODYNAMICS.

322. Hydrodynamics treats of the motions of liquids and of the forces which they exert upon the bodies when their action is applied.

323. The particles of a fluid on escaping from an orifice possess the same velocity as if they had fallen freely in *vacuo* from a height equal to that of the fluid surface above the centre of the orifice. This is known as *Torricelli's theorem.*

324. The principal deductions from the Torricellian theorem are—

1st. The velocity of an escaping fluid depends upon the depth of the orifice beneath the surface and is independent of the density of the liquid.

2nd. The velocity of efflux from an orifice is as the square root of the height of the fluid surface above the centre of the orifice.

NOTE.—Since all bodies falling *in vacuo* from the same height acquire the same velocity, density has no effect in increasing the velocity of a liquid escaping from an orifice in the side or in the bottom of a vessel. Thus water, alcohol, and mercury, will all flow with the same rapidity; for though the pressure of the mercury is $13\frac{1}{2}$ times greater than that of water, it has $13\frac{1}{2}$ times as much matter to move.

325. When a liquid flows from an orifice in a vessel which is not replenished but the level of which continually descends, the velocity of the escaping liquid is uniformly retarded, being as the decreasing series of odd numbers 9, 7, 5, 3, &c., so that an unreplenished reservoir empties itself through a given aperture in twice the time the same quantity of water would have required to flow through the same aperture had the level been maintained constantly at the same point.

326. The quantity of fluid discharged from a given aperture in a given time is found by multiplying the area of the aperture by the velocity of the escaping liquid.

NOTE.—Experiments do not agree with this theory as regards the quantity of liquid discharged. The whole subject has been carefully investigated by Bossut, and he has shown that

Actual discharge : *Theoretical discharge* :: ·62 : 1 or as 5 : 8.

Hence the theoretical discharge must be multiplied by $\frac{5}{8}$ to obtain the true quantity.

Fig. 27.

This discrepancy arises from the fact that the escaping jet diminishes in diameter just after leaving the vessel, forming what is known as the *vena contracta* or contracted vein. The minimum diameter of the vein is found at a distance about equal to half the diameter of the aperture at $c\,c'$ Fig. 27. This effect arises from the fact that just above the orifice the lateral particles of fluid move as well as the descending portions.

If the jet of liquid be thrown upwards at an angle of from 25° to 45° the vein retains the diameter of the aperture, but if thrown at an angle greater than 45° its section increases.

327. *Let* $Q =$ *the quantity discharged in* 1 *second,* $a =$ *area of aperture,* $h =$ *height of fluid level above the centre of the orifice,* $g =$ *accelerating force of gravity, and* $v =$ *velocity.*

Then Art. 266 $v = \sqrt{2gh},$ (I) $Q = a\sqrt{2gh},$ (II) $a = \dfrac{Q}{\sqrt{2gh}},$ (III)

and $h = \dfrac{Q^2}{2ga^2}.$ (IV.)

NOTE.—Since $g = 32$, $2g = 64$, and $\sqrt{2g} = 8$, formulas I, II and III become respectively $v = 8\sqrt{h}$, $Q = 8a\sqrt{h}$, and $a = \dfrac{Q}{8\sqrt{h}}.$

328. An *adjutage* is a short tube, either cylindrical or conical, placed in an orifice to increase the flow. If the vein passes through the tube without wetting the interior walls, the flow is not modified, but if the liquid adhere, i. e., wet the walls, the *vena contracta* is dilated and the flow increased.

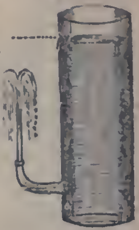
329. A cylindrical adjutage with length not greater than four times its diameter increases the flow one-third.

330. A conical adjutage, converging towards the exterior, augments the flow more than a cylindrical adjutage —its effect upon the vein varying with the angle of convergence.

331. A conical adjutage diverging towards the exterior is still more efficient and may be such as to render the flow three or four times as great as the actual flow from an orifice of the same diameter in a thin wall and 1·5 times greater than the theoretical flow.

332. As the velocity of a liquid escaping through an orifice is the same as it would have acquired in falling freely *in vacuo* through a space equal to the distance of the orifice below the level of the liquid, it follows that a jet of water spouting upwards should rise to the level of the liquid in the reservoir. In practice, however, the spouting jet never reaches this height owing to certain disturbing forces, namely:—

Fig. 28.



1st. Friction in the conducting tube in part destroys the velocity.
2nd. Atmospheric resistance.
3rd. The returning water falls upon that which is rising and thus tends to stop its ascent.

NOTE.—The height to which the liquid spouts is increased by:
1st. Having the orifice very small in comparison with the conducting tube.
2nd. Piercing the orifice in a very thin wall; and
3rd. Inclining the jet a little so as to avoid the returning water.

EXAMPLE 402.—With what velocity does water issue from a small aperture at the bottom of a vessel filled to the height of 100 feet?

SOLUTION.

Formula 1 $v = 8\sqrt{h} = 8\sqrt{100} = 8 \times 10 = 80$ feet per second. *Ans.*

EXAMPLE 403.—What quantity of water will be discharged in one minute from an aperture of half an inch in area—the height of the water in the vessel being kept constant at 10 feet above the centre of the orifice?

SOLUTION.

Here $a = \frac{1}{2}$ square inch $= \frac{1}{288}$ of a square foot,
The cubic feet discharged in 1 second $= 8\,a\sqrt{h}$.
Cubic feet discharged in 1 minute $= 60 \times 8a \times \sqrt{h} = 60 \times \frac{8}{288} \times \sqrt{10}$ $= 60 \times \frac{1}{36} \times 3{\cdot}162 = 5{\cdot}27$ cubic feet = the theoretical quantity, and $5{\cdot}27 \times \frac{5}{8} = 3{\cdot}29$ cubic feet = true quantity.

EXAMPLE 404.—What must be the area of an orifice in the side of a vessel in order that 40 cubic feet of water may issue per hour—the water in the reservoir being kept constantly at the level of 20 feet above the centre of the aperture?

SOLUTION.

Here $Q = \frac{40}{60 \times 60} = \frac{1}{90}$ of a cubic foot, and since this is only $\frac{5}{8}$ of the theoretical quantity, $Q = \frac{8}{5}$ of $\frac{1}{90} = \frac{4}{225}$ of a cubic foot. Also $h = 20$

Then formula III, $a = \frac{Q}{8\sqrt{h}} = \frac{\frac{4}{225}}{8\sqrt{20}} = \frac{\frac{4}{225}}{35{\cdot}776} = \frac{5}{10062}$ of a foot $\frac{120}{1677}$ of an inch. *Ans.*

Example 405.—An upright vessel 16 feet deep is filled with water, and just contains 15 cubic feet. Now, if a small aperture $\frac{1}{4}$ of an inch in area be made in the bottom, in what time will the vessel empty itself?

SOLUTION.

Here $h = 16$ ft., $a = \frac{1}{4}$ of an inch, and $Q = 15$ cubic feet.
Hence the theoretical quantity $= 15 \times \frac{8}{5} = 24$ cubic feet.
Then velocity at commencement $= 8\sqrt{h} = 8\sqrt{16} = 32$ ft.
Quantity discharged in one second $= 32 \times \frac{1}{576} = \frac{32}{576} = \frac{1}{18}$ of a cubic foot.
Time required to discharge 24 cubic feet $= 24 \div \frac{1}{18} = 432$ seconds.

But, Art. 324, when a vessel empties itself, the time required to discharge a given quantity of water is double that requisite for discharging the same quantity when the level is maintained.
Hence time $= 432 \times 2 = 864$ seconds $= 16{\cdot}4$ minutes. *Ans.*

EXERCISE.

406. With what velocity does water issue from a small aperture in the side of a vessel filled to the height of 25 feet above the centre of the orifice? *Ans.* 40 feet per second.

407. With what velocity does water flow from a small aperture in the side of a vessel filled with water to the height of 17 feet above the centre of the orifice?
Ans. 32·984 feet per second.

408. In the last example, if the water flows into a vacuum, what is its velocity? *Ans.* 56 feet per second.

Note.—Since the pressure of the atmosphere is equal to that of a column of water 32 feet high, the effective height of the column of water is $17 + 32 = 49$ feet.

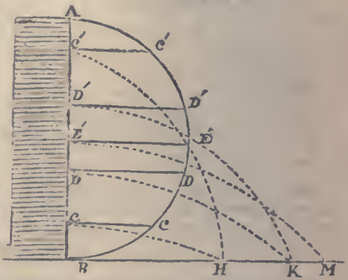
409. How much water is discharged per minute from an aperture having an area of $\frac{1}{5}$ of an inch—the surface of the fluid being kept constant at 36 feet? *Ans.* $2\frac{1}{2}$ cubic feet.

410. What must be the area of the aperture in the bottom of a vessel in order that 90 cubic feet of water may issue per hour—the level of the water in the vessel being constantly kept at 20 feet above the centre of the orifice?
Ans. 161 or about $\frac{4}{25}$ of an inch.

411. A vessel contains 20 cubic feet of water, which fills it to the depth of 30 feet—now, if an aperture having an area of $\frac{7}{8}$ of an inch be made in the bottom of the vessel, in what ime will it empty itself? *Ans.* 2 min. $30\frac{1}{2}$ sec.

333. When water spouts from several apertures in the side of a vessel, it is thrown with the greatest random from the orifice nearest the centre, the jet issuing from the centre will reach a horizontal distance equal to the entire height of the liquid, and all jets equally distant from the centre will be thrown to an equal horizontal distance.

Fig. 29.



NOTE,...Let VA be a vessel filled with water, having its side AB perpendicular to the horizontal plane BM. On AB describe the semicircle BDA. Bisect AB in C and in AB take any points D and D' equally distant from E, also C and C' equally distant from E. Draw also CC, DD, EE, &c., perpendicular to AB and produce to the circumference ABC. Then if small orifices be pierced in the side of the vessel at C', D', E', D', and C, the liquid from E will spout to twice $EE' = AB = BM$; the liquid from C or C' will spout to H = twice CC or CC' and that from D or D will reach K = twice DD or $D'D'$.

334. The horizontal distance to which the liquid spouts under these circumstances may be found as follows:

Let H = height of water above horizontal plane, d = perpendicular let fall to the orifice from the circumference $A\ E'\ B$, and h = height of orifice above the horizontal plane. Then (Euclid iii. 35)

$$d^2 = h\ (H - h) \text{ and hence } d = \sqrt{h\ (H - h)}$$

Thus if the Reservoir in Fig. 29 be 20 feet in height and be filled with water, and the apertures C, E, and D be respectively 4, 10 and 15 feet above the plane $B\ M$; then the segments of $A\ B$ are respectively 4 and 16, 10 and 10, and 15 and 5 feet, and the randoms will be respectively $2 \times \sqrt{4 \times 16}$, $2 \times \sqrt{10 \times 10}$ and $2 \times \sqrt{15 \times 5}$, i. e. $2 \times \sqrt{64}$ or 16 ft. $2 \times \sqrt{100}$ or 20 ft. and $2 \times \sqrt{75}$ or 17·32 ft.

335. When water flows in any bed, as in the channel of a river or in a pipe, the velocity becomes constant when the length of the bed bears a large proportion to its sectional area. Thus, in pipes of more than 100 feet in length, or in rivers whose course is unopposed by natural obstacles, the velocity of the body of the stream is the same throughout. When this occurs the liquid is said to be *in train.*

336. The velocity of the liquid flowing in a pipe or channel is not the same in every part of its *section*, being greatest in the centre of the section of the pipe or in the middle of the surface of the stream.

NOTE 1.—This arises from the friction exerted against the fluid by the interior surface of the pipe or the banks of the stream. In a stream, on account of the middle part having the greatest velocity, the surface is always more or less convex.

NOTE 2.—The velocity of a stream may be determined in three different ways:—

1st. An open tube bent at right angles is placed in a stream with one of its legs opposed to the current and the other branch vertical—the velocity of the stream is measured by the height to which the water rises in the vertical leg.

2nd. A float is thrown into the stream and the time occupied by it in passing over a known distance observed.

3rd. The convexity of the surface may be measured by a levelling instrument, and its velocity thus determined.

337. *To find the velocity of efflux, and hence the quantity of water discharged in a given time from a reservoir of given height through a pipe of given length and diameter :—*

Let $d =$ *diameter of pipe,* $l =$ *length,* $h =$ *height, and* $v =$ *velocity.*

Then all the dimensions being in feet, $v = 48\sqrt{\left\{\frac{hd}{l+54d}\right\}}$

NOTE.—This is the formula of M. Poncelet, and is regarded as strictly accurate.

WATER WHEELS.

338. Water is frequently made to drive machinery by its weight or momentum exerted on a vertical water-wheel.

339. There are three varieties of vertical water-wheels, viz.: the *undershot*, the *overshot*, and the *breast wheel.*

Fig. 30.

BREAST WHEEL.

Fig. 31.

UNDERSHOT WHEEL.

Fig. 32.

OVERSHOT WHEEL.

NOTE.—The mode in which the water is made to act on these is represented in Figs. 30, 31, 32. It will be observed that the undershot wheel is moved by the momentum of the water—the breast wheel and overshot wheel by its weight aided by its momentum. An overshot wheel will produce twice the effect of an undershot wheel,—the dimensions, fall, and quantity of water being the same. The breast wheel is found to consume twice the quantity of water required by an overshot wheel to do the same work.

340. In all water-wheels the greatest mechanical effect is produced when the velocity of the water is $2\frac{1}{2}$ times that of the wheel.

341. To find the horse power of a vertical water-wheel:—

Let b *= breadth of stream in feet,* d *= depth of stream,* v *= mean velocity in feet of stream per minute,* h *= height of fall,* s *= weight of one cubic foot of water, and* m *= modulus of the wheel.*

Then horse power $= \frac{mbdvsh}{33000.}$

EXAMPLE 412.—A water wheel is worked by a stream 6 feet wide and 3 feet deep, the velocity of the water is 22 feet per minute, and the height of the fall 30 feet, required the horse power of the wheel, the modulus being ·7.

SOLUTION.

$$H.P. = \frac{mbdvsh}{33000} = \frac{6 \times 3 \times 22 \times 30 \times 62{\cdot}5 \times {\cdot}7}{33000} = 15{\cdot}75\ Ans.$$

EXAMPLE 413.—What is the horse power of a water wheel worked by a stream 2 feet deep, 7 feet wide, and having a velocity of 33 feet per minute—the fall being 10 feet and modulus of the wheel ·6 ?

SOLUTION.

$$H.P. = \frac{mbdvsh}{33000} = \frac{.6 \times 7 \times 2 \times 33 \times 62{\cdot}5 \times 10}{33000} = 4.25.\ Ans.$$

EXAMPLE 414.—A water reservoir is 100 feet in height, supplies water to a city by a pipe 10000 feet in length and 6 inches in diameter, what is the velocity per second and what quantity will be discharged in 24 hours?

SOLUTION.

Here $h = 100$, $l = 10000$, and $d = \frac{1}{2}$.

Then Art. 330, $v = 48\sqrt{\left\{\frac{hd}{l + 54\,d}\right\}} = 48\sqrt{\left\{\frac{100 \times \frac{1}{2}}{10000 + 54 \times \frac{1}{2}}\right\}} = 48\sqrt{\frac{50}{10027}}$

$= 3{\cdot}36$ feet per second $=$ velocity.

Quantity discharged in 1 second $= 3{\cdot}1416 \times (\frac{1}{4})^2 \times 3.36$.

Quantity discharged in 24 hours $= 3{\cdot}1416 \times \frac{1}{16} \times 3{\cdot}36 \times 60 \times 60 \times 24 =$ 57001·1904 cubic feet. *Ans.*

EXERCISE.

415. A water-wheel is worked by a stream 4 feet wide and 3 feet deep, the velocity of the water is 29 feet per minute, the fall 20 feet; required the horse power of the wheel, its modulus being ·56? *Ans.* 7·38

416. A water wheel is worked by a stream 2 feet deep and 4 feet wide, and having a velocity of 50 feet per minute, the fall is 33 feet and the modulus ·84, how many cubic feet of water per hour will this wheel raise from the depth of 44 feet? *Ans.* 15120.

417. A water-wheel is worked by a stream 4 feet wide and 3 feet deep, the velocity of the water being 15 feet per minute, and the fall 27 feet, how many gallons of water per hour will this wheel raise to a height of 80 feet, the modulus being ·8? *Ans.* 18225 gallons.

418. A water reservoir 80 feet in height supplies water to a city through a pipe 1742 feet in length and 4 inches in diameter, what is the velocity of the water per second, and how many gallons will it deliver in 10 hours?

Ans. 115925·04 gallons.

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342. The Turbine is a horizontal water-wheel having a vertical axle. It revolves entirely submerged, and is of all varieties of water-wheels the most economical and powerful. It was invented by M. Fourneyron in 1827, but has since been much modified in form and greatly improved. The water enters at the centre of the wheel, descends in its vertical axis and is delivered by a great number of curved guides so as to strike the buckets in a direction nearly tangential to the circumference of the wheel. The buckets are also curved in the direction required to give the machine the greatest possible amount of efficiency. The water having expended its force escapes from the wheel in a direction corresponding very nearly with the radii.

343. Turbine wheels may be divided into high pressure and low pressure machines.

344. High pressure turbines are such as are worked by a high fall of water, and are adapted to hilly countries and deep mines, where the height of the fall may be made to compensate for the smallness of the volume of water.

345. Low pressure turbines are employed where a large stream of water possesses but little fall; they are said to produce powerful effects with a head of water of but nine inches.

346. A committee of investigation appointed by the French Academy of Sciences, and consisting of Arago, Prony, and others, gave the following report on these wheels:—

I. Turbines are equally applicable to high or low falls of water.

II. Their effective work is from 70 to 78 per cent. of the work applied. (Turbines, made by Boyden of Boston, have given 88 per cent. of the work applied.)

III. They work without much loss of power at velocities both above and below that required to produce the maximum effect.

IV. They will work without appreciable loss at a depth of from 4 to 6 feet beneath the surface of water.

NOTE.—In another modification of these horizontal wheels the water is made to apply at the periphery of the wheel. Many varieties are patented and highly spoken of as to their effective performances.

CHAPTER IX.

THEORY OF UNDULATIONS.

347. All undulations or waves have their origin in vibratory or oscillatory movements imparted to the molecules of the solid, liquid, or gaseous body in which the undulations occur.

348. Undulations are of two kinds,

1st. Progressive undulations.

2nd. Stationary undulations.

349. Progressive undulations are such as are produced by the vibratory movement passing from the particles first affected to those next them, and the oscillation being thus communicated successively from particle to particle, the wave advances with a progressive movement.

As familiar illustrations of this kind of undulatory movement, we may mention the waves produced on water by the wind, or by casting a stone on its surface, and those produced in a cord made fast at one end, by smartly shaking the other end up and down. In the latter case, a wave-like movement is observed to pass from the hand to the fast end of the cord, and then a similar wave returns to the hand.

NOTE.—It must be carefully remembered that although the wave advances, the particles by whose vibration it is produced have themselves no progressive motion, but a mere oscillatory movement up and down like that of a pendulum. Thus in the case of the cord, the particles of matter that compose the cord do not themselves recede from the hand and advance to it. And that there is no progressive forward movement in the particles of water producing water-waves is evidenced by the fact, that a float placed on the surface of the water simply rises and falls with the wave but does not move forward with it.

350.—Stationary undulations are such as are produced when all the particles of a body are made to assume and to complete these vibrations at the same times.

Thus when a cord or a wire is stretched between two fixed points, and is made to vibrate by drawing it at the middle from its rectilinear position, it recovers its normal condition after performing a series of undulations in which all the particles of the cord or wire take part.

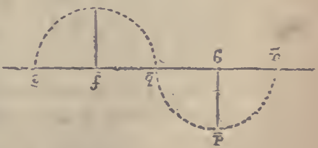
351. In every undulation certain parts are to be distinguished as follows:—

The curve *a d b e c*, is called an *undulation wave.*

The part *a d b*, is the *phase of elevation.*

The part *b e c*, is the *phase of depression.*

Fig. 33.

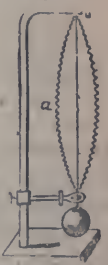


The distance $a\ c$ is the *length* of the wave.
The distance $d\ g$, is its *height*.
The distance $f\ e$, is its *depth*.
Twice $d\ g$, or $f\ e$, is its *amplitude*.

352. The vibration of solid bodies may be conveniently considered under the heads of cords, rods, planes, and masses. Stretched strings, wires or other linear solids, are susceptible of three kinds of vibration, viz:

1st. Transverse vibrations.
2nd. Longitudinal vibrations.
3rd. Torsional vibrations.

Fig. 34.



Thus if a cord be secured at one end and held stretched by a weight attached to the other as in Fig. 34, then
1st. Upon drawing the string to one side and suddenly letting it go the vibrations which it makes and which are represented by the dotted lines are at right angles to the axis of the cord and are called *transverse* vibrations.
2nd. If the ball B be raised a little and suddenly dropped, it will continue for some time advancing and receding from its original position, the cord performing a series of *longitudinal* vibrations.
3rd. If the ball be turned round its vertical axis several times, and then let go, the cord will for some time continue to twist and unwist, thus performing a series of *torsional* vibrations.

353. In transverse vibrations the *time of vibration* is the time occupied in passing from a to b, that is, in making one complete movement from side to side.

354. The vibrations of stretched cords, wires, &c., are always isochronous (See Art. 310), and are governed by the four following laws:

I. *The tension being the same, the number of vibrations of a cord varies inversely as its length.*
II. *The tension and length being the same, the number of vibrations in cords of the same material, is inversely as their diameters.*
III. *The number of vibrations of a stretched cord is proportional to the square root of the force of tension, i, e., the stretching weight.*
IV. *All other things being equal, the number of vibrations of different cords is inversely proportional to the square root of their densities.*

Thus by the first law, equally stretched cords of the same material vibrate more rapidly in proportion as they are shortened. For example, if several strings of cat-gut be equally stretched and their lengths are represented by the numbers 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{11}$. &c., their vibrations in the same space of time will be represented by the numbers 1, 2, 3, 5, 7. 11, &c.

By the second law, if we have cords or wires of the same material of equal length and tension, but having a thickness represented by the numbers 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c., then the number of their vibrations in the same unit of time, will be respectively represented by the numbers 1, 2, 3, 4, 5, &c.

By the third law, if we have a cord of wire stretched with a certain degree of force and therefore vibrating with a certain rapidity in order to double, triple, quadruple, &c., the rapidity of the vibrations, we shall have to strain the cord to four, nine, sixteen, &c., times its original tension.

By the fourth law, if we have two cords of the same tension, length and diameter, but one formed of cat-gut having a spec. grav. or density of 1, and the other formed of copper having a spec. grav. or density of nearly 9, the former will vibrate about three times as rapidly as the latter.

355. When a stretched cord as *a b*, Fig. 35, is fastened at each extremity and also temporarily fixed at two intermediate points *d* and *c*, the segment, *a d*, *d c* and *c b* may be thrown in stationary vibrations of equal amplitude. Upon now loosening the points *d* and *c* it will be found that these points remain at rest although the other parts of the cord are in rapid vibration. These *points of rest* are called *nodal* points(Lat. *nodus* "a knot,") and occur wherever the phases of elevation and depression in such a vibratory line intersect each other.

Fig. 35.

NOTE.—The *nodal* points of a vibratory line or rod may be experimentally determined by small rings of paper which remain fixed on these points, but are thrown off from all others. A stretched line or rod may be thrown into a series of stationary vibrations by drawing a violin bow across it in different places.

356. A rod like a stretched string, may vibrate either transversely, longitudinally, or torsionally, and is subject in its vibrations to the following laws:

I. *The vibrations are isochronous.*

II. *The transverse vibrations vary in number inversely as the square root of the length of the rod.*

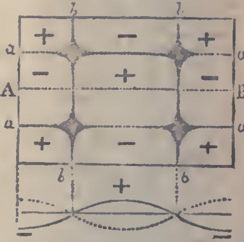
III. *Longitudinal vibrations vary in number inversely as the lengths of the rods no matter what may be their diameters or the forms of their transverse sections.*

IV. *Torsional vibrations of rods, of the same material, vary in number directly as their thickness and inversely as their lengths.*

357. An elastic plate may be made to vibrate by fastening it in a vice either by the corner or by the centre and drawing a violin-bow across its edge.

358. In a vibrating plate certain lines exist which are always at rest and which are hence called *nodal* lines. They correspond to the nodal points in strings or rods, and if we regard the plate as being composed of a number of rods then we may consider the nodal lines will be made up of their nodal points.

Fig. 36.



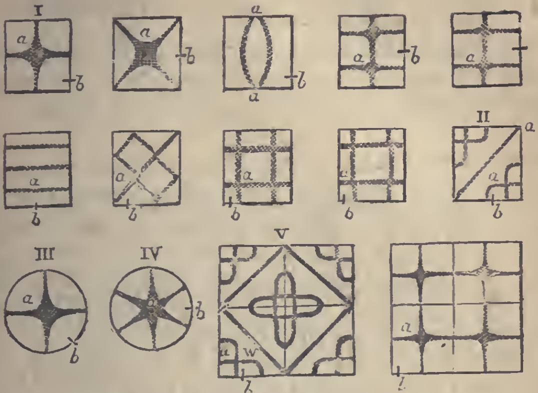
The plate is divided by the nodal lines into *internodal spaces*, the adjacent spaces being always in opposite phases as shown in Fig. 36, where the sign + indicates the phase of elevation, and the sign — the phase of depression.

359. The nodal lines vary in number and position according to the form of the plate, its size, its elasticity, the rapidity of the vibrations, the mode in which they are produced, the point by which the plate is fixed, &c. Their position may be determined by scattering sand or colored powder on the plate and vibrating by means of violin-bow,—the sand is thrown off the internodes and arranges itself along the nodal lines forming the so called *nodal figures* or *acoustic figures.*

360. Nodal figures have a great variety in their form but are generally very symmetrical. Several hundred have been figured. The accompanying illustration represents a few of those obtained on square and circular plates.

The plates are supposed to be fastened in a vice at the point *a*, and the violin-bow drawn over the edge at the point *b*. In figure III the finger is placed on the edge of the plate at a point 45° from *b*, in IV at a point 60° or 30° or 90° from *b*. In V the finger is placed at *w*.

Fig. 37.



361. The vibrations of elastic plates are performed according to the following laws:—

I. *The number of vibrations is independent of the breadth of the plate.*

II. *The number of vibrations is proportional to the thickness of the plate.*

III. *The thickness being the same, the number of vibrations varies inversely as the square of its length.*

NOTE.—The plate is supposed to be, in each case, composed of the same substance.

UNDULATIONS IN LIQUIDS.

362. Undulations in a liquid are caused by the vibratory movement of its molecules in such a manner that each particle describes a vertical circle, about the spot in which it may chance to be, revolving in the direction of the advancing wave. This rotating movement among the particles is progressively carried to the contiguous particles, so that different atoms will be describing different parts of their circular path at the same moment. Thus some will be at the point of highest elevation, forming the crest of the wave, others at the point of lowest depression forming the trough, and others at intermediate points.

The diameter of the vertical circle described by a single particle is called the amplitude of the wave, and is, in the case of ocean waves, often as much as 20 feet. It has been ascertained by experiment that a liquid is not disturbed by the undulations on its surface, to a depth greater than about 175 times the amplitude of the wave.

363. Progressive undulations striking against a solid surface are reflected and the angle of reflection is always equal to the angle of incidence. It follows from this law that:—

1st. If the wave be linear, i. e., if its crest is at the right angles to its course and it meets a plane surface perpendicularly it will be reflected back in the same path, and if it meet the plane surface at an angle of 80°, 40°, 30°, &c., it will be reflected on the other side of the perpendicular at the same angle.

2nd. The rays of a wave originating in one focus of an elliptical vessel are all reflected to the other focus.

3rd. The rays of a wave propagated in the focus of a parabola are all reflected in parallel lines.

4th. A line or wave impinging on a parabola has all its rays reflected to the focus of the parabola.

5th. If two parabolas face each other with their axes coincident, a system of circular waves originating in one focus will be followed by a corresponding system having the other focus for their centre.

6th. When the rays of a circular wave impinge at right angles upon a plane surface they are reflected so as to form a circular wave having the same degree of curvature but in the opposite direction.

364. When two systems of waves originating in different centres meet, they either combine or interfere and their interference may be either complete or partial.

I. When two waves meet in the same phase, i. e., so that their elevations and depressions coincide, they combine and form a new wave having an amplitude equal to the sum of the amplitudes of the combining waves.

II. If the two waves of equal amplitude meet in opposite phases, i. e., so that the depression of the one coincides with the elevation of the other they interfere, both waves disappear, and the liquid surface becomes perfectly horizontal.

III. If two waves of unequal amplitude meet in opposite phases they partially interfere, and the resulting wave has an amplitude equal only to the difference between the amplitude of the meeting waves.

UNDULATIONS IN ELASTIC FLUIDS.

365. All elastic fluids, such as atmospheric air, are subject to surface undulations such as occur in liquids; and these surface undulations are governed by the same laws.

366. When an elastic fluid is compressed, and the compressing force is suddenly removed, the fluid expands beyond its normal dimensions, it then contracts, a second time expands, and thus continues, for some time, to oscillate alternately on each side of its original volume. The pulsations or waves which are thus engendered in elastic fluids differ from the surface waves produced in the same fluid, and also from the waves that are peculiar to water and other non-elastic fluids in the following particulars:

1st. Aërial waves or undulations consist in the alternate rarefaction and condensation of the air or other gas, and are hence called *waves of rarefaction and waves of condensation;* and

2nd. Aërial waves are always spherical in form.

367. Aërial waves are influenced with respect to interference and combination by the same general laws as govern the surface wave of liquids (See Art. 364), but we must bear in mind that the term *rarefaction* corresponds to phase of elevation, and *condensation* to phase of depression.

CHAPTER X.
ACOUSTICS.

368. Acoustics (Greek "Akouō" "to hear,") treats of sounds, their cause, production and nature, and the laws by which they are governed.

11

369. Sounds are sensations arising from impressions made upon the auditory nerve by waves or undulations in the surrounding medium.

370. All bodies producing sound are in a state of more or less rapid vibration; and these vibrations, impinging upon the atmosphere or other elastic medium, produce in it a series of undulations of condensation and rarefaction.

The vibrations of a stretched cord producing sound may be perceived by placing the finger on it; the vibrations of a sonorous plate by scattering sand upon it, &c.

371. The intensity of the sound produced by a vibrating sonorous body depends chiefly upon two circumstances:—

1st. The density of the surrounding medium, and
2nd. The rapidity of the vibration of the sonorous body.

372. Sound is not propagated at all in a vacuum, and the sound produced in atmospheric air by a vibrating sonorous body is much more intense than that produced in hydrogen and other gases of less density than air. On the other hand, solid bodies, vapors, water and other liquids of greater density than air, transmit sound with increased energy.

Sounds are not only much louder but can be heard to a much greater distance in water and solids than in air. Thus if the ear be applied to one end of a long beam of wood and the least tapping noise or even the scratch of a pin be applied to the other—the sound is distinctly perceived by the ear. The report of cannon is said to have been distinctly heard to the distance of 250 miles by applying the ear to the solid earth. If the ear be placed under the surface of water, and two pebbles be knocked together, the sound conveyed to the ear is very loud, and it is said that if a cannon be fired close to a body of water in which a person has his head immersed, the report is sufficient to destroy his sense of hearing.

373. All sounds travel, in the same medium, with the same velocity, whatever may be their pitch or their strength.

Were it not for this property of sound—the notes produced by the musical instruments of an orchestra would be discordant, except to those in the immediate neighbourhood of the performers.

NOTE.—It has lately been satisfactorily shown that in the case of sounds differing very widely in intensity this is not strictly true,—very intense sounds travel rather more rapidly than others.

374. The velocity of sound in atmospheric air varies: 1st. With the temperature, decreasing about $1\frac{1}{10}$ ft. per second, for every degree Fahr. the temperature is lowered.

2nd. With the velocity and direction of the wind.

NOTE.—The intensity of a sound as heard at a distance is much modified, but its velocity is not affected by the condition of the air as to its being clear or foggy, the barometric pressure great or small, the sky clear or cloudy.

375. Accurate experiments have determined the velocity of sound in atmospheric air at a temperature of 60° F., to be 1118 feet per second.

376. The velocity of sound in vapors and gases at 32° F., has been determined from calculation by Dulong to be as follows:

Carbonic acid,.........	860	feet per second.
Alcohol vapor,.........	862	" "
Oxygen,...............	1040	" "
Olefiant gas,...........	1030	" "
Air,..................	1092	" "
Carbonic oxide,........	1105	" "
Water vapor,..........	1347	" "
Hydrogen,..	4163	" "

377. The following table gives the results of experiments made upon the velocity of sounds in liquids and solids:

In Water,	sound	travels at	rate of	4708	feet	per second.
Tin,	"	"	"	8385	"	"
Cast Iron,	"	"	"	11609	"	"
Copper,	"	"	"	13416	"	"
Wood,	"	"	"	16770	"	"

NOTE.—That is, in water sound travels 4½ times as fast as in air; in wood about 15 times, and in metals from 7 to 12 times as fast.

378. The distance to which sound may be propagated depends upon the following circumstances:

I. The greater the intensity of the sound the greater the distance to which it will travel.

II. The denser the air or other conducting medium the greater the distance to which the sound will travel.

III. In atmospheric air the distance to which the sound will travel is much influenced by the condition of the air as regards winds, &c.

379. It has been experimentally ascertained that the following sounds may, under ordinary circumstances, be heard at the annexed distances:

The human voice in the open air,.................	700 ft.
The marching of a company of soldiers at night,..	2500 ft.
The marching of a company or squadron of cavalry,	3000 ft.
The report of a musket,.........................	3000 ft.

NOTE.—Lieut. Foster conversed with a man, in frosty weather, across the harbor of Fort Bowen, a distance of 1¼ miles. Dr Young states that the watchword "All's well" has been heard from Old to New Gibraltar, a distance of 10 miles. The cannonading of a sea fight between the English and Dutch in 1672 was heard at Shrewsbury, a distance of 200 miles.

The cannonading at the siege of Antwerp in 1832 was heard in the mines of Saxony, a distance of 320 miles.

The noise produced by the volcanic eruption in Tomboro in Sumbawa was heard at a distance of 900 miles.

380. When two series of sonorous undulations encounter each other in opposite phases of vibration, they interfere, and if the sound produced by each separately are equal, the interference will be complete, they will destroy each other and produce silence.

381. The phenomenon of interference of sonorous waves so as to produce silence may be conveniently shown in the following manner:

Take two tuning forks of the same note, fasten to one prong of each a small disc of card board half an inch in diameter, and make one fork rather heavier than the other by loading it with a little sealing wax at the end. Also take a glass jar about ten inches in height and two inches in diameter. Then make one of the forks vibrate, and holding it just above the mouth of the glass vessel as seen at *d*, Fig. 38, carefully pour in water till the air in the jar vibrates in unison with the fork, and the result will be the production of a prolonged uniform and clear sound without stop or cessation. When either fork is made to vibrate and is held *alone* over the jar, we obtain a uniform sound, but when both are made to vibrate and are at the same time held over the mouth of the jar there arise a series of sounds alternating with a series of silences, this alternation continuing as long as the forks are vibrating.

The explanation is simply that the long waves arising from one fork overtake the shorter waves produced by the other, and alternately interfere and combine with them.

The destruction of sonorous waves by interference may also be observed by holding a vibrating tuning fork about a foot from the ear and gradually turning it round. When the prongs are equally distant from the ear a note is heard, but when one is more distant than the other partial or complete interference takes place, and the sound dies out in part or altogether.

382. Sound waves are reflected upon striking any solid or liquid surface according to the laws enumerated in Art. 363.

NOTE.—A certain portion of the sound enters the second medium and undergoes refraction.

383. An echo is a sound reflected by a surface sufficiently distant to allow a short space of time to intervene between the direct and the reflected sounds in order that these may not be confounded.

384. The ear cannot distinguish one sound from another unless there be an interval between the two at least one-ninth of a second. In one ninth of a second sound travels 124 feet (1118÷9) so that a perfect echo cannot exist unless there be at least 62 feet (half of 124) between the ear and the reflecting surface.

If a sentence be repeated in a loud voice at the distance of 62 feet from a reflecting wall the last syllable will be distinctly echoed; if at the distance of 124 ft. the last two syllables ; if at the distance of 186 ft. [62×3] the last three syllables, &c.

If the reflecting wall is at a less distance than 62 ft. from the speaker, reflected sounds blend with the emitted so as to prolong and strengthen them. This is expressed by the term resonance. Hangings, draperies, carpets, &c., about a room tend to smother or stifle the sound, as they are bad reflectors. A crowded audience has a similar affect—increasing the difficulty of speaking by presenting non-reflecting surfaces.

If a person stands 1118 feet from a reflecting surface and articulates loudly at the rate of four syllables per second, the echo will repeat the last eight syllables clearly; because the sound will require two sounds to travel to the reflecting surface and back to the ear, and in two seconds he gives utterance to eight syllables.

385. Repeating or multiplying echoes are those that repeat the same sound several times. Such echoes commonly occur where parallel walls or other obstacles are placed opposite each other at a sufficient distance apart to reflect the sonorous undulations alternately from side to side.

In a multiplying echo each repetition is less loud because the reflected wave is always more feeble than the direct wave, so that intensity is lost by each reflection until the sonorous undulations become incapable of conveying any impression to the ear.

386. *Remarkable Echoes.*—There is an echo at Verchères between two towers that repeats a word thirteen times.

At Adermach in Bohemia there is an echo which repeats seven syllables three times distinctly.

At Lurleyfels on the Rhine there is an echo which repeats seventeen times.

At Belvidere, Alleghany County, N. Y., there is an echo between two barns which repeats distinctly a word of one, two or three syllables eleven times.

At Woodstock in England there is an echo which repeats seventeen times during the day and twenty times during the night.

In the Villa Simonetta near Milan, there is an echo which repeats a sharp sound thirty times.

The celebrated ancient echo of Metelli at Rome is reputed to have been capable of repeating the first line of the Æneid containing fifteen syllables eight times distinctly.

387. Whispering galleries are so called because a whisper uttered at one point may be distinctly heard in some other remote locality although quite inaudible in all other positions. They are generally domed or are of an ellipsoidal form—the point of utterance and the focus of reflection being the two foci of the ellipse (compare Art. 363).

The most remarkable whispering galleries in the world are the following:

The gallery beneath the dome of St. Paul's Cathedral in London.

The Gothic vault of the Cathedral at Gloucester.

A Church at Girgenti in Sicily in which a whisper near the door is distinctly heard at the remote end of the church 200 feet distant.

The Grotto di Favella, at Syracuse [supposed to be the celebrated Ear of Dionysius.]

The dome of the rotunda of the Capitol at Washington, &c.

388. The speaking trumpet is an instrument designed to enable the human voice to be heard to a great distance. Its efficacy is due to the fact that the confined column of air is made to vibrate in unison with the voice, and hence the pulsations that impinge upon the exterior air, have a greater energy and give rise to sonorous waves of greater intensity.

It has been satisfactorily shown by Hassenfratz that the old explanation by reflection of the rays of sound is inadmissible. This is proved by the fact that the power of the instrument is not impaired by lining its interior with linen, a very bad reflector, or by making the trumpet in the form of a cylinder provided with a bell-shaped extremity. The shape of the extremity exerts an unexplained influence upon the action of the instrument. The sound emitted through the trumpet is increased in all directions, i. e., not merely in the quarter to which it is pointed.

389. The ear trumpet is designed to enable partially deaf people to distinguish sounds more distinctly. It acts upon the principle that the portion of the sonorous wave that enters the large end of the instrument imparts its energy to portions of air smaller and smaller and consequently causes it to vibrate or pulsate with more intensity as it approaches the ear.

We have an illustration of something of the same kind of concentration when we attach a weight to a string and cause it to wind rapidly round the finger; the revolutions become more rapid as the string shortens.

It was formerly customary to explain the action of the ear trumpet upon the principle of reflection of the rays or waves of sound. This explanation is disproved by the fact that so long as the extremity remote from the ear is much larger than that applied to that organ, it makes but little or no difference what may be the shape of the trumpet. It likewise makes no difference whether the interior surface is rough or polished.

CHAPTER XI.

MECHANICAL THEORY OF MUSIC.

390. Noise is the effect of a series of sonorous undulations produced by unequal or irregular vibrations.

The report of a gun, the crack of a whip, the rumble of a train of cars, or of a carriage on a stone road, &c., are familiar examples of noises.

391. Musical sounds are the result of sonorous waves produced by equal or regular vibrations.

392. Every sound has three distinct qualities distinguishable by the ear, viz.:

I. The pitch or tone.

II. The intensity.

III. The quality or timbre.

393. The tone or pitch of a sound is high or low, and depends upon the rapidity of the vibratory movement producing the sonorous undulation. The more rapid the vibrations are, the higher will be the pitch of the note.

394. The intensity or loudness of the sound depends upon the amplitude of the vibrations which produce the sonorous wave, or what amounts to the same thing, upon the degree of condensation produced in the middle of the sonorous undulation.

NOTE.—A sound may maintain the same pitch, i. e., the same length of wave, and yet vary in intensity.

395. The quality or timbre of a sound is that property or peculiarity which enables us to distinguish it from all other sounds of the *same pitch and intensity*.

Thus if a flute, a piano, a violin, and a clarionet, all sound a note of the same pitch and with the same intensity, we can readily distinguish the sound produced by each.

396. Sounds produced by the same number of vibrations per second, are said to be in *unison*.

397. A *melody* is a succession of single musical sounds which bear to each other such simple relations as are readily perceived by the ear, and which consequently produce an agreeable impression.

398. A *chord* consists of two or more melodious sounds produced simultaneously.

399. A *harmonized passage* consists of a succession of chords following one another in melodious order.

To a cultivated ear a ring of bells is musical or noisy according as its tones are musical or unmusical intervals; it is harmonious or discordant according as the intervals are concords or dissonances; it will be "cheerful" or "sad" according as the intervals producing the concordances are major or minor.

400. The instruments used for determining the number of vibrations performed by a sonorous body giving a tone of definite pitch, are the *Sirene* and *Savart's toothed wheel.*

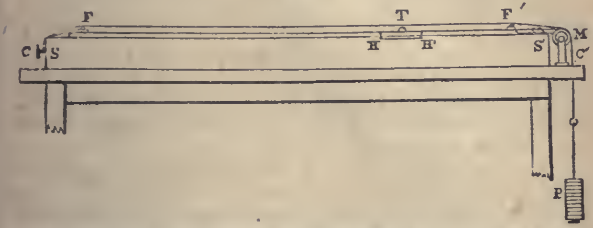
The essential parts of the Sirene are a brass tube about 4 inches in diameter, terminating in a smooth brass plate which has about twenty small holes pierced obliquely near its circumference. A second thick plate having the same number of equidistant holes but pierced obliquely in the reverse direction, is supported just above the first plate in such a manner as to revolve with extreme ease. At the upper extremity of the vertical axis which bears the second plate, there is an endless screw, which acts upon a counter, like that on a gas meter. The lower part of the tube bearing the first plate, terminates in an air chamber which is kept filled with uniformly compressed air by a double acting bellows. When a current of air arrives from the bellows it passes through the holes in the first plate, and in escaping through the second plate imparts to the latter a rotary motion. As the upper plate revolves the avenues of escape for the compressed air are rapidly cut off and renewed, and consequently when the plate revolves regularly and with sufficient rapidity, sonorous undulations are produced in the exterior air by the minute puffs of wind that escape at uniform intervals through the plates—the sound increasing in acuteness as the velocity of the revolving disc becomes greater. The rapidity of the revolution is governed by the degree of pressure to which the air in the chamber is subjected.

Savart's toothed wheel consists of a large wheel connected by means of an endless band with the axis of a smaller toothed wheel, the cogs of which are made to touch in succession a small tongue or slip of metal, thus causing it to vibrate. The number of revolutions made by the toothed wheel is recorded by an attached system of clock-work; and the number of vibrations made by the tongue is found of course by simply multiplying the number of revolutions by the number of teeth in the wheel. It is perhaps, unnecessary to remark that the more rapid the revolution of the wheel the more rapid is the vibration of the tongue, and consequently the higher the pitch of the note produced. Each tooth causes the tongue to make two movements, i. e., one down and the other up, each of these is called a single vibration, and the two together a double vibration.

Both the Sirene and Savart's toothed wheel act upon the recognised principle that two sounds are in unison when they are produced by the same number of vibrations per second. The instrument is made to revolve more or less rapidly till it is brought in unison with the sound experimented on when the rate of vibration is at once obtained from the dial face.

401. The Monochord or Sonometer is an instrument used to study the transverse vibrations of cords, and hence the relation that subsists as regards number of vibrations, &c., between the several notes of the musical scale.

Fig. 39.



The monochord consists of a thin wooden case SS′ above which a metallic wire or a cord of catgut FTF′ is stretched over the pulley M by the weight P. A moveable bridge HH′ can be placed at any desired point between the fixed bridges F and F′. The weight P is commonly adjusted so that the string or wire when vibrating its whole length shall give the note C.

402. If the whole length of cord vibrating produces the note C, it is found by experiment that when $\frac{8}{9}$ of its length vibrate, the note D is produced; $\frac{4}{5}$ of its length vibrating give the note E, &c.; and since (Art.) the number of vibrations varies inversely as the length of the string, these fractions inverted give the number of vibrations necessary to produce the notes D, E, &c., as compared with C. The following table gives the relative lengths of cord producing the notes of the common diatonic scale, and the relative numbers of vibrations per second belonging to them.

	C *do*	D *re*	E *mi*	F *fa*	G *sol*	A *la*	B *si*	C *do*
Relative length of cords........	1	$\frac{8}{9}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{8}{15}$	$\frac{1}{2}$
Relative number of vibrations..	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

403. It is common to indicate the different scales in use by means of indices attached to the various notes. Thus the fundamental C which corresponds to the highest sound of the base, is represented by C^1, the successive higher octaves by C^2, C^3, C^4, &c., and the successive lower octaves by C^{-1}, C^{-2}, C^{-3}.

404. The absolute number of vibrations corresponding to any given note can easily be determined by setting the Sirene or Savart's wheel in unison with it. It has been thus ascertained that the fundamental C is produced by 128 simple vibrations per second, and by multiplying this successively by $\frac{8}{3}$, $\frac{5}{4}$, $\frac{4}{3}$, $\frac{3}{2}$, $\frac{5}{3}$, &c., we obtain the number of vibrations for the other notes as given in the following table :—

Notes	C	D	E	F	G	A	B	C
Absolute number of simple vibrations per second.	128,	144,	160,	$170\frac{2}{3}$,	192,	$213\frac{1}{3}$,	240,	256

405. The number of vibrations corresponding to the several notes of any superior gamut, is found by multiplying the above numbers by 2, 4, 8, &c., and for the inferior gamut by dividing by 2, 4, 8, &c.

Thus $A^3 = 213\frac{1}{3} \times 4 = 853\frac{1}{3}$ simple vibrations $= 426\frac{2}{3}$ complete vibrations.

$C^{-3} = 128 \times 4 = 512$ " " $= 256$ " "
$A^{-2} = 213\frac{1}{3} \div 4 = 53\frac{1}{3}$ " " $= 26\frac{2}{3}$ " "
$C^{-2} = 128 \div 4 = 32$ " " $= 16$ " "

NOTE.—There is a slight difference in the actual number of vibrations producing a particular note as performed in different cities. Thus the number of vibrations required to produce A^3 varies as follows:—

Theoretical number	$426\frac{2}{3}$.
Orchestra of Berlin Opera	$437\frac{1}{3}$.
Opera Comique, Paris	$427\frac{2}{3}$.
Academie de la musique, Paris	$431\frac{1}{3}$.
Italian Opera, (1855)	449.

The General Musical Congress which met in London in the year 1860 to consider the propriety of adopting a uniform musical pitch, fixed upon the number 528 complete vibrations for C^3, 440 for A^3.

The commission recently appointed in France have recommended $C^3 = 522$; $= A^3 = 435$. In the report submitted by this committee the following pitches were discussed :—

Handel's Tuning Fork (c. 1740)	A at 416	= C at $499\frac{1}{5}$.
Theoretical Pitch	A at $426\frac{2}{3}$	= C at 512.
Philharmonic Society (1812-42)	A at 433	= C at 518.
Diapason Normal (Paris, 1859)	A at 435	= C at $522\frac{2}{5}$.
Stuttgard Congress (1834)	A at 440	= C at 528.
Italian Opera, London, (1859)	A at 455	= C at 546.

Pianofortes for private purposes are usually tuned somewhat below concert pitch, so that A^3 is produced by about 420 complete vibrations per second.

406. The length of a sonorous vibration is found by dividing 1120 feet, the velocity of sound per second, by the number of vibrations made per second, in order to produce the note. The following table gives the wave-length of the C of different scales:—

Notes.	Simple Vibrations per second.	Wave-lengths in feet.
C^{-3}	16	70
C^{-2}	32	35
C^{-1}	64	$17\frac{1}{2}$
C^{1}	128	$8\frac{3}{4}$
C^{2}	256	$4\frac{3}{8}$
C^{3}	512	$2\frac{3}{16}$
C^{4}	1024	$1\frac{3}{32}$
C^{5}	2048	$\frac{35}{64}$
C^{6}	4096	$\frac{35}{128}$

407. Interval indicates how much one sound is higher than another in pitch, and is of course greater or less as the difference in the number of vibrations, producing the two sounds, is greater or less.

408. Musical intervals are named thirds, fourths, fifths, &c., from the position of the higher note counting upwards from the lower, according to the following table, in which the first line gives the name of the note; the second line, the number of its vibrations, as compared with the first note; the third line, the name of the interval; and the fourth line, the interval obtained by dividing each note by that which precedes it.

C	D	E	F	G	A	B	C′	D′	E′	F′	G′	A′	B′	C″
1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2	$\frac{18}{8}$	$\frac{10}{4}$	$\frac{8}{3}$	$\frac{6}{2}$	$\frac{10}{3}$	$\frac{30}{8}$	4
1st.	2nd.	3rd.	4th.	5th.	6th.	7th.	8th.	9th.	10th.	11th.	12th.	13th.	14th.	15th.
	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{10}{16}$

NOTE.—The second line of this table must be interpreted thus:—In order to produce the second note D, 9 vibrations must be made in the same time required by 8 vibrations giving the first note C; in order to obtain the third note E, 5 vibrations must be in time required by 4 of the first note C and so on; or, taking 24 the least common denominators of the fractions, while the vibrations producing the first note C number 24, those required to produce the successive following notes will be 27, 30, 32, 36, 40, 45, 48, 60, 72, 80, 90, and 96.

409. In examining the foregoing table two points must be carefully noted.

I. There are but *three* different intervals between the successive notes of the scale, viz., $\frac{9}{8}$, $\frac{10}{9}$ and $\frac{16}{15}$.

II. These intervals occur in the same order in each successive octave.

The interval $\frac{9}{8}$, being the largest interval found in the scale, is called a *major tone;* $\frac{10}{9}$ is called a *minor tone*, and $\frac{16}{15}$ is called a *semitone*, although it is greater than one-half of either a major or a minor tone.

NOTE.—The interval $\frac{16}{15}$ is frequently spoken of as a *diatonic semitone;* the difference between a major tone and the diatonic semitone, *i. e.*, $\frac{9}{8} - \frac{16}{15} = \frac{7}{120}$, is called a *chromatic semitone;* the difference between a minor tone and the diatonic semitone, *i. e.*, $\frac{10}{9} - \frac{16}{15} = \frac{2}{45}$, is called a *grave chromatic semitone;* the difference between a major tone and a minor tone, *i. e.*, $\frac{9}{8} - \frac{10}{9} = \frac{1}{72}$, is called a *comma.*

410. The following table exhibits all the intervals that occur in comparing the notes of the common scale two and two:

C..D=F..G=A..B	$= \frac{9}{8}$, a major tone.
D..E=G..A	$= \frac{10}{9}$, a minor tone $= \frac{80}{81}$ of $\frac{9}{8}$.
E..F=B..C	$= \frac{16}{15}$, diatonic semitone $= \frac{24}{25}$ of $\frac{10}{9}$ or $\frac{24}{25}$ of $\frac{80}{81}$ of $\frac{9}{8}$.
C..E=F..A=G..B	$= \frac{5}{4}$, a major third.
E..G=A..C=B′..D′	$= \frac{6}{5}$, a minor third $= \frac{24}{25}$ of $\frac{5}{4}$.
D..F	$= \frac{80}{81}$ of a minor third $= \frac{80}{81}$ of $\frac{6}{5} = \frac{24}{25}$ of $\frac{80}{81}$ of $\frac{5}{4}$.
C..F=D..G=E..A=G..C′	$= \frac{4}{3}$, a perfect fourth.
A..D′	$= \frac{27}{20}$, a sharp fourth $= \frac{81}{80}$ of $\frac{4}{3}$.
F..B	$= \frac{45}{32} = \frac{135}{128}$ of a perfect fourth, $= \frac{25}{24}$ of $\frac{81}{80}$ of $\frac{4}{3}$.
C..G=E..B=F..C′=G..D′ =A..E′	$= \frac{3}{2}$, a perfect fifth.
D..A	$= \frac{40}{27} = \frac{80}{81}$ of a perfect fifth.
B..F	$= \frac{64}{45}$, an inharmonious interval.
C..A=D..B=F..D′=G..E	$= \frac{5}{3}$, a perfect sixth.
A..F′=B..G′	$= \frac{8}{5}$, a minor fifth $= \frac{24}{25}$ of $\frac{5}{3}$.
F..D′	$= \frac{54}{32}$, an inharmonious interval.
C..B=F..E′	$= \frac{15}{8}$, a seventh, an inharmonious interval.
D..C′=G..F′=B..A′	$= \frac{16}{9}$, a flattened seventh, more harmonious than the perfect seventh.
E..D′=A..G′	$= \frac{9}{5}$, a minor seventh $= \frac{24}{25}$ of $\frac{15}{8}$
C..C′	$= \frac{2}{1}$, an octave.

411. Compound chords consist of three or four notes whose vibrations have a simple numerical relation to one another, and which taken together two and two, produce harmony.

The *Perfect Major Accord* consists of the three notes C, E and G, whose vibrations are to each other as the numbers 4, 5, and 6, and which compared together two and two give the relations $\frac{5}{4}$, $\frac{6}{5}$ and $\frac{6}{4}$. The *Perfect Minor Accord* consists of the three notes E, G and B, whose vibrations are as the numbers 10, 12 and 15, and which give the relations $\frac{6}{5}$, $\frac{5}{4}$ and $\frac{3}{2}$.

NOTE.—The intervals of the perfect minor differ from those of the perfect major accord only in their order.

412. Any tone whatever in the common scale or any pitch whatever, may be adopted as the basis of another similar scale, provided means be employed to preserve the same relative intervals between the successive notes. When a piece of music is thus changed from one scale into another it is said to be *transposed*, and the process is called the *transposition of scales.*

413. In the transposition of scales it is found necessary to introduce additional notes, in order to maintain the relative intervals between the successive notes. Such additional notes are called *sharps* (♯) and *flats* (♭) according as the tone corresponding to any given note is raised or lowered.

414. When these new notes are interpolated in every full tone (major or minor) of the diatonic scale, the result is a series of twelve intervals in the octave, forming what is known as the *chromatic scale.*

415. Temperament is an artifice by means of which the introduction of an inconveniently large number of additional notes into the scale is prevented. In the transformation of scales it is assumed that every note may be raised or lowered by a diatonic semitone $\frac{16}{15}$, but in order actually to raise and lower each tone by that amount, we would require a very great number of new notes. To prevent this such notes as C♯ and D♭ are regarded as identical, though in reality they differ from one another slightly, and are played differently on stringed instruments, as the harp and violin, by skilful players. For practical purposes musical instruments such as pianos, organs, &c., are tuned so as to divide the octave into 12 equal intervals called *chromatic semitones of equal temperament.* It follows from this that all musical intervals except octaves, as played on instruments, differ more or less from absolute purity; thus in the following table it will be seen that the minor semitone and the major thirds are all too sharp, and the major semitones, the minor thirds and the fifths, are all too flat.

	True value.	Value in equal temperament.
Minor semitone......	$\frac{25}{24} = 1{\cdot}042$	$\sqrt[12]{2} = 1{\cdot}060$
Major semitone......	$\frac{16}{15} = 1{\cdot}067$	
Minor third.........	$\frac{6}{5} = 1{\cdot}200$	$\sqrt[12]{2^3} = 1{\cdot}189$
Major third.........	$\frac{5}{4} = 1{\cdot}250$	$\sqrt[12]{2^4} = 1{\cdot}260$
Fifth	$\frac{3}{2} = 1{\cdot}500$	$\sqrt[12]{2^7} = 1{\cdot}498$

Another mode of explaining what is meant by temperature is the following:—

While the key note makes one vibration, the major third must make $\frac{5}{4}$ vibrations, the major third of this note must make $\frac{5}{4}$ of $\frac{5}{4} = \frac{25}{16}$ vibrations, and the major third of this last note $\frac{5}{4}$ of $\frac{5}{4}$ of $\frac{5}{4} = \frac{125}{64}$. This last note does not accord perfectly with the true octave which is 2 or $\frac{128}{64}$. If then we keep the octave pure we cannot retain the purity of the thirds, and the same occurs with respect to the fifths. In order therefore, to retain the octaves pure we have to raise or lower the thirds and fifths somewhat above or below their normal tone. This balancing or suffering the note to float somewhat over or under its proper tone is called *tempering*.

The subject of temperament is a very extensive one, and the student is directed for its full investigation to any of the standard works on music.

NOTE.—If the ear were more sensitive than it is, it would be so unpleasantly affected by the impurity of the thirds and fifths, as almost to preclude any enjoyment from musical performances.

416. When two sounds not in unison, are produced at the same time, alternation of strength and feebleness are perceived. These alternations follow each other at regular intervals, and are called *beats*. The nearer the vibrations agree in rapidity, the longer is the interval between the beats; when the unison is perfect no beat occurs; and when the vibrations differ widely in rapidity they produce merely an unpleasant rattle.

417. The *tuning fork* or *diapason* is a two pronged steel fork of peculiar form, by means of which we can produce an invariable note. It is commonly formed to give A^3, corresponding to 428 vibrations per second, but may be made so as to give any other note of the gamut. It is much used as a standard in tuning instruments, or striking the key note in vocal music, &c.

NOTE.—The note given by the diapason is much strengthened by mounting it on a box of thin wood open at one end.

MUSICAL INSTRUMENTS.

418. Musical instruments may be for the most part divided into—

I. Wind instruments.

II. Stringed instruments.

III. Instruments of which the essential part is a stretched membrane.

419. Wind instruments are sounded either by an embouchure like the flute, organ, pipe, flageolet, &c., or by reeds as in the Jew's-harp, clarionet, melodeon, horns, trumpets, trombones, &c.

420. Stringed instruments are all compound — the sounds produced by the vibrating string being strengthened by elastic plates of wood or metal and inclosed portions of air to which the cords transmit their own vibrations. Stringed instruments are played—

I. By a bow as in the violin.

II. By percussion as in the piano, or

III. By twanging as in the harp.

421. The third class of musical instruments includes drums, tambourines, &c. Drums are of three kinds,—the small drum or common regimental drum, which is a brass cylinder having both heads covered with membrane but beaten only at one end; the base or double drum which is much larger, and which is beaten at both heads; and the kettle drum which is a hemispherical copper vessel supported on a tripod, and having its head covered with vellum.

The kettle drum has an opening in the metallic case to equalise the vibrations.

422. In all wind instruments the sounds are produced by throwing the column of air contained in tubes into vibration. The pitch of the sound produced depends upon:—

1st. The length of the tube containing the air.

2nd. The position and dimensions of the embouchure.

3rd. The manner of imparting the primary motion to the air.

The difference of quality belonging to the notes given by pipes of different materials is due most likely to a feeble vibration of the sides of the tube.

423. Sonorous vibrations are produced in tubes—

I. By blowing obliquely into the open end of the tube as in the Pandean pipe.

II. By casting a current of air into an embouchure near the closed end of the tube as in mouth pipes.

III. By thin laminæ of metal or wood placed at the end of the tube and which vibrate as the current of air sweeps past. These laminæ are called *reeds.*

IV. By the lips acting as reeds.

V. By a small burning jet of hydrogen gas.

424. The laws that govern the vibration of air in tubes were investigated by Bernoulli in 1782. He divides all tubes into two classes.

1st. Tubes having the extremity opposite the mouth closed.

2nd. Tubes open at both extremities.

For tubes with the end remote from the mouth closed he gives the following laws:—

I. The same tube may produce different sounds and in this case the number of vibrations will be to each other as the odd numbers 1, 3, 5, 7, 9, &c.

II. In tubes of unequal length sounds of the same order correspond to the number of vibrations and these are in inverse ratio to the length of the tube.

III. The column of air vibrating in a tube is divided into equal parts which vibrate separately and in unison—the open orifice being always in the middle of a vibrating part, and the length of a vibrating part equal to the length of a wave corresponding to the sound produced.

For tubes open at both ends the foregoing laws prevail, with the following modifications :

I. The sounds produced are represented by the natural numbers 1, 2, 3, 4, 5, 6, 7, &c.

II. The fundamental sound of a tube open at both extremities is always the acute octave of the same sound in a tube closed at one extremity.

III. The extremities of the tube are in the middle of a vibrating part.

CHAPTER XII.

THE ORGANS OF VOICE AND HEARING.

THE ORGANS OF VOICE.

425. Many animals have the power of producing sounds, and as a general rule those that are endowed with a voice have also the organ of hearing well developed. Man alone, however, possesses the gift of speech, i. e., the power of giving to the tones he utters a variety of definite articulate sounds.

426. The vocal apparatus of man consists of the following parts :—

I. The Thorax, which, by means of the intercostal muscles and the diaphragm, acts as a bellows in producing a current of air for the production of sounds.

II. The Windpipe, which is a long tube carrying the air from the lungs to the organs more immediately concerned in forming the voice.

III. The Larynx (Adam's apple), which is the musical organ of the voice, and corresponds to the mouth-piece of a musical instrument.

IV. The Pharynx—a large funnel-shaped cavity at the top of the larynx or at the back of the mouth, which, by varying in form and tension, modifies the tones of the voice.

V. The mouth and nasal passages, which correspond to the upper part of an organ tube, and throw the vibrations into the air.

427. The larynx is composed of the *hyoid* bone, and its attached cartilages, viz., the two *thyroid*, which form the sides and front of the larynx, and which constitute the prominence known as the pomum Adami—the *cricoid*, which is ring-shaped, and rests upon the top of the trachea, and the two *arytenoid*, at the back of the larynx and between the two thyroid cartilages. The arytenoid cartilages are movable to a small extent by means of several muscles attached to them.

428. THE CORDÆ VOCALES, or vocal cords, are two ligaments, of elastic fibrous substance, which extend from the arytenoid cartilages behind to the thyroid cartilages in front. The ligaments meet in front, but are somewhat separated behind; so that when at rest they form an opening in the interior of the larynx shaped like a V; but by the drawing together of the arytenoid cartilages, the open end may be closed in such a manner that the two vocal cords shall touch one another along their entire length, and the aperture be completely closed. The opening between the vocal cords is called the *rima glottidis*, or fissure of the glottis.

429. The membrane which lines the interior of the larynx doubles so as to form a second pair of folds just above the vocal cords. The space between these is much wider than that between the vocal cords, and is covered during the act of deglutition by a valve-like flap called the *epiglottis*. The space between the upper and lower pair of ligaments is called the *glottis*, or the *ventricles of the larynx*.

430. Except during the production of vocal sounds the arytenoid cartilages are wide apart, and the vocal cords wrinkled and plicated; but while the organs of voice are in action, the *rima glottidis* is so narrowed that the

sides, rather than the edges of the vocal cords, are in contact, and while the ligaments are thus in contact, the air passing throught the larynx sets them in vibration, somewhat like the reed of a clarionet or the tongue of a trumpet, and the result is the production of a sound. The pitch of the sound depends of course on the rapidity of the vibration, and this is governed by the lèngth and the degree of tension of the vocal cords. The vocal cords are tightened or relaxed by means of the muscles that act on the thyroid and arytenoid cartilages.

NOTE.—Some physiologists regard the return of the glottis in producing sound as analogous to that of a bird call.

431. One of the most remarkable circumstances in connection with the organs of voice and their action is the perfect precision with which the will can determine the degree of tension of these ligaments. Their average length while in repose is in the adult male about $\frac{73}{100}$ of an inch, and in the adult female $\frac{51}{100}$; and when stretched to their utmost capacity their length is only $\frac{93}{100}$ in the male, and $\frac{63}{100}$ in the female. The extreme difference of length is therefore about $\frac{1}{5}$ of an inch in the male, and about $\frac{1}{8}$ of an inch in the female. The average compass of the cultivated voice is about two octaves or 24 semitones, and as a practiced singer can produce at least 10 distinct intervals within each semitone, the range of the voice may be said to be 240 notes. Each of these 240 notes corresponds to a different degree of tension of the vocal cords; and as the utmost limits of tension are $\frac{1}{5}$ of an inch in the male and $\frac{1}{8}$ of an inch in the female, it follows that in man the difference in length of the vocal cords required to pass from one interval to another will not be more than $\frac{1}{1200}$ of an inch, and in woman not more than $\frac{1}{2000}$ of an inch.

NOTE.—It is said that the celebrated vocalist Madame Mara was able to sound 100 different notes within each interval of the diatonic scale, and as the compass of her voice was 20 tones, the whole number of notes she could sound was 2000, all of course comprised within the extreme variation of $\frac{1}{8}$ of an inch. It may hence be said that she was capable of determining with precision the contraction of the vocal cords to the $\frac{1}{16000}$ of an inch.

432. The larynx is about the same size, and consequently the vocal cords are about the same length in both sexes up to the age of 14 or 15 years; however from that time it rapidly increases in size in the male, but remains stationary in the female. It is owing to this greater length of the vocal cords that the pitch of a man's voice is lower than that of a woman, or of a girl, or of a boy.

433. The difference of *timbre* or quality in different voices, appears to be chiefly due to the difference of flexibility and smoothness in the cartilages of the larynx. Women and children have these cartilages smooth and flexible, and hence their voice is smooth; men, on the contrary, have cartilages which are harder, and sometimes completely ossified, and hence the roughness—the want of flexibility of their voices.

434. The loudness of the voice depends principally, upon the force with which the air is expelled from the chest, but in part also the resonance produced by the other parts of the larynx and the neighboring cavities.

NOTE.—In the *howling monkeys* of South America there are several hollow pouches which open from the larynx, and one in the hyoid bone (which is greatly enlarged). The voice of this variety of monkey is said to be louder than the roar of the lion.

435. Voices are divided by musicians into the following classes:—

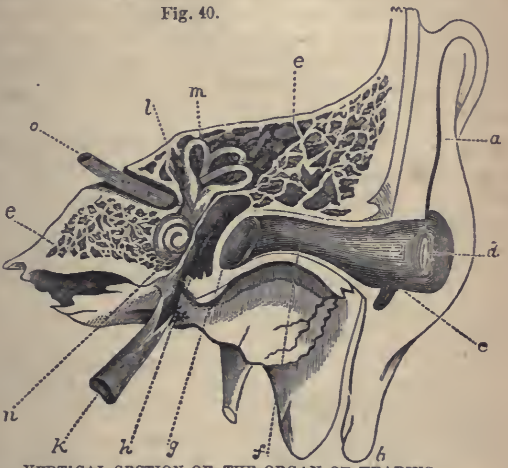
		Double vibrations per second made by vocal cords.
Female voices,	Soprano.	From 1056 to 264.
	Mezzo-Soprano.	" 930 " 220.
	Contralto.	" 704 " 176.
Male voices,	Tenor.	" 528 " 132.
	Baritone.	" 352 " 110.
	Base.	" 220 " 82½

NOTE.—In speaking, the range of the voice is limited to about half an octave, in singing, to about two octaves. Occasionally a person may be met with who has cultivated his voice, so as to reach through three octaves. The entire range of the human voice, taking both male and female together, may be said to be about four octaves.

436. Birds have a true larynx *which* is often very complex, and which is placed at the *lower* extremity of the trachea, just where it branches into the bronchial tubes. The upper end of the trachea opens into the pharynx by a mere slit. Birds in which the true or lower larynx is absent, are necessarily voiceless. In the cat the upper and lower vocal cords are almost equally developed, and hence the variety and range of its voice. The horse and ass have only two vocal cords. The sounds produced by insects are caused by percussion or by rubbing the horny sheaths of their wings or legs together, or by the rapid vibrations of their wings or by the contraction and expansion of their air tubes, which forces the air through their orifices so as to make it whistle.

THE ORGANS OF HEARING.

Fig. 40.



VERTICAL SECTION OF THE ORGAN OF HEARING.

437. The organ of hearing in man is composed of three parts, viz. :—

I. The External Ear or Pinna.

II. The Middle Ear or Tympanum.

III. The Internal Ear or Labyrinth :—

438. The External Ear consists of two parts.

I. The Pinna or Pavilion (*abc*), also called the *ala* or wing and the auricula.

II. The *Meatus Auditorius,* or auditor canal (*d*). Both the pinna and the auditory canal are cartilaginous in structure, but are abundantly supplied with vessels, and hence it is that the ears tingle and redden even with very slight mental emotion. The pinna collects the waves of sound and directs them inward to the tympanum, through the auditory canal. The precise purpose served by the numerous prominences and depressions on the pinna is not satisfactorily known. The auditory canal is about an inch long in the adult, and extends from the pinna to the drum or tympanum. Its entrance is guided by hairs; and further to prevent the intrusion of insects, there is a very bitter and somewhat fetid wax secreted along its entire length.

NOTE.—Many animals possess the ability to turn their external ears in different directions, the better to collect the soniferous waves; and it is worthy of remark, that beasts of prey can turn their ears forward with most facility, while timid animals commonly keep their ears directed backwards so as to guard against the approach of danger from behind, their eyes serving to keep them warned of what is going on in front.

439. The Middle Ear or *tympanum* or *tympanic cavity* is a somewhat hemispherical cavity, about half-inch diameter; it is placed in the temporal bone, extending from the drum to the vestibule, and is filled with air. The parts of the middle ear are:—

I. The *Membrana Tympani*, or drum of the ear.

II. The Eustachian Tube.

III. The four small bones of the ear.

The membrana tympani is placed obliquely across the inner end of the auditory canal. It is thin and oval, and is placed at an angle of 45°, its outward plane looking forwards and downwards.

The Eustachian tube is a membraneous canal leading from the middle ear downwards and forwards into the pharynx, with which it communicates by means of a valvular opening that is generally closed. It gives exit to the mucus which forms in the middle ear, and also permits the entrance of air into the tympanic cavity; when closed by a cold, it causes partial deafness.

Fig. 41.

The ossicles of the tympanum are four small bones which connect the membrana tympani with the fenestra ovalis. They, are shown magnified in the Fig. and are named from their shapes; the *malleus* or hammer, *m*, the *incus* or anvil, *i*, the *os orbiculare* or round bone *o*, (the smallest bone in the body), and the *stapes* or stirrup, *s*. The handle *h* of the malleus is fastened to the membrana tympani, and the base of the stapes to the membrane covering the *fenestra ovalis*. The bones are joined to one another in the position represented in the figure, and are capable of slight movement by means of attached muscles.

440. The labyrinth or internal ear has its channels excavated in the petreous bone, the hardest of any in the body. It consists of the following parts:—

I. The Vestibule.

II. The semi-circular Canals.

III. The Cochlea.

The vestibule (*l*) is a chamber formed in the petreous bone. Various branches of the auditory nerve and of arteries pass into it, and it is connected with the tympanic cavity by means of two orifices which are covered with membranes, viz., the *fenestra ovalis* or oval window (*o*, Fig. 42), and the *fenestra rotunda* or round window (*r*, Fig. 42).

The semicircular canals (*x*, *y*, and *z*) are three in number, passing from and returning into the vestibule in the upper posterior part. They are placed at right angles to one another, and are each filled by a membraneous canal of the same shape, containing fluid.

Fig. 42.

The *cochlea* (snail shell), *n* Fig. 40 and *k* Fig. 42, is a spiral cavity, having the exact form of a snail's shell, the convolutions making just two turns and a half around a central pillar. The canal is divided into two passages by a partition (the lamina spiralis), which runs its entire length. These passages do not communicate except at the top, where there is a small opening through the partition; at the lower end (corresponding to the mouth of the snail shell) they terminate separately, one with the tympanic cavity by means of the *fenestra rotunda*, and the other opens freely into the vestibule.

441. The whole interior of the labyrinth is lined by a delicate membrane, on which the auditory nerve is minutely distributed. Small looped fibrils of this nerve depend from the lamina spiralis, and float in the watery liquid which fills the cochlea as well as the other parts of the labyrinth.

442. The functions of the different parts of the ear are as follows:—

I. The waves of sound are collected in the pinna or external ear, are directed through the auditory canal, and striking upon the membrana tympani throw it into vibration.

II. The chain of small bones connecting the *membrana tympani* with the membrane that cover the *fenestra ovalis* receives the vibrations from the drum or membrana tympani, and transmits them across the tympanic cavity through the fenestra ovalis into the vestibule.

III. The vibrations which are thus excited in the fluid which fills the vestibule, semicircular canals and cochlea, are received by the expanded filaments of the auditory nerve, and the sensation of sound is thus transmitted to the brain.

443. Careful experiments have determined the following principles with regard to the transmission of vibrations from one medium to another, and a due consideration of these will explain the arrangement of membranes, and solids, and fluids in the ear.

I. Atmospheric vibrations lose much of their intensity when transmitted directly either to solids or liquids.

II. The intervention of a membrane greatly facilitates the communications of vibrations from air to liquids.

III. Atmospheric vibrations are readily communicated to a solid, if the latter be attached to a membrane so placed that the vibrations of the air act upon it.

IV. A solid body fixed in an opening by a border membrane so as to be movable, communicates sonorous vibrations from air on one side to water or other similar fluids on the other, much better than solid media not so constructed.

444. The peculiar functions of the semi-circular canals and of the cochlea, are not very well known. As the former are always placed at right angles to each other, occupying the position of the bottom and two sides of a cube, it has been supposed that they enable us to judge of the direction of sound: it is also deemed highly probable by physiologists that the cochlea serves to give us the idea of the pitch of sounds.

445. According to Savart the most grave note the ear is capable of appreciating is formed by from seven to eight complete vibrations per second. When fewer vibrations are made per second, they are heard as distinct sounds, i. e., do not produce a note. The most acute note appreciated by the ear is produced by 365000 complete vibrations per second.

NOTE.—The interval *la* is said to be heard by rapidly moving the head from side to side, owing to the motion of the small bones of the ear.

446. The mechanism of hearing is not equally complicated in all classes of animals.

Birds have the internal and middle ear constructed on the same general plan as man, but the external ear is merely a circlet of feathers.

Reptiles have no external ear, and in many cases no middle ear. The fluid in the vestibule is rendered milky in color, owing to the abundance of minute crystals of phosphate of lime.

Fishes have no external or middle ear, but simply a membraneous vestibule situated in the skull, and surrounded by semi-circular canals from one to three in number.

The ear of the mollusk is simply a sack filled with liquid, and having the auditory nerve expanded upon its inner surface.

The position of the organs of hearing in insects is not very well known; but some, as the grasshopper, have the ear no longer in the head but in the legs.

MISCELLANEOUS PROBLEMS.

1. What must be the length of a pendulum in the latitude of Canada in order to vibrate once in 5 seconds?
2. In a lever the arm of the power is 7 feet long and the arm of the weight 2 feet 7 inches; with this instrument what power will sustain a weight of 743 lbs.?
3. In a hydrostatic press the sectional areas of the cylinders are to one another as 1427 is to 3, and the force pump is worked by means of a lever whose arms are to one another as 27 to 2. Now if a power of 87 lbs. be applied to the extremity of the lever, what upward pressure will be exerted against the piston in the larger cylinder?
4. A cannon ball is fired vertically with an initial velocity of 600 feet per second; it is required to find—
   1st. How far it will rise.
   2nd. Where it will be at the end of the 7th. second.
   3rd. In how many seconds it will again reach the ground.
   4th. What will be its terminal velocity.
   5th. In what other moment of its flight it will have the same velocity as at the end of the 5th second of its ascent.
5. A water-wheel is worked by means of a stream 4 feet wide and $3\frac{1}{2}$ feet deep, the water having a velocity of 27 feet per minute and falling from a height of 36 feet, how many strokes per minute will it give to a forge hammer weighing 7000 lbs.,—the vertical length of the stroke being 4 feet?
6. In a differential wheel and axle the radii of the axles are $3\frac{1}{4}$ and $3\frac{3}{7}$ inches, and a power of 7 pounds sustains a weight of 1000, what is the radius of the wheel?
7. How far may an empty vessel capable of sustaining a pressure of 159 lbs. to the square inch be sunk in water before breaking?
8. In a screw the pitch is $\frac{3}{16}$ of an inch, the power lever 9 feet 2 inches long and the weight is 44000 lbs., what is the power?
9. How many units of work are expended in raising 70 cubic feet of water to the height of 83 feet?
10. The piston of a low pressure steam engine has an area of 360 inches and makes 13 strokes of 7 feet each per minute, the pressure of the steam on the boiler being 40 lbs. to the square inch. Required the horse power of the engine.

11. Through how many feet will a power of 7 lbs., moving through 120 feet, carry a weight of 29 lbs.?

12. A train weighing 75 tons is drawn along an inclined plane with a uniform velocity of 40 miles per hour, assuming the inclination of the plane to be $\frac{1}{7}$ in 100, and taking friction and atmospheric resistance as usual, what is the horse power of the engine :—

1st. If the train is ascending the plane?

2nd. If the train is descending the plane?

13. If a body weighing 7 lbs. at the surface of the earth be carried to a distance of 30,000 miles from the earth, what will be its weight?

14. With what velocity per second will water flow from a small aperture in the side of a vessel, the fluid level being kept constantly 12 feet above the centre of the orifice?

15. In a hydrostatic bellows the tube has a sectional area of $1\frac{1}{2}$ inches, the area of the board is 37 inches, and the tube is filled with water to the height of 28 feet, what upward pressure is exerted against the board of the bellows?

16. In a differential wheel and axle the radii of the axles are $1\frac{3}{8}$ and $2\frac{1}{7}$ inches, the radius of the wheel is 40 inches, what power will sustain a weight of 8,700 lbs.?

17. A clock is observed to lose 17 minutes in 24 hours, how much must the pendulum be shortened in order that it may keep correct time?

18. At what height will the mercury stand in a barometer at an elevation of 3·5 miles?

19. An upright flood gate of a canal is 17 feet wide and 13 feet deep, the water being on one side only and level with the top; required the pressure :—

1st. On the whole gate.

2nd. On the lowest three-fifths of the gate.

3rd. On the middle three-fifths of the gate.

4th. On the upper four-elevenths of the gate.

5th. On the lowest five-twelfths of the gate.

20. A piece of stone weighs 23 oz. in air and only 14·7 oz. in water; required its specific gravity.

21. Through how many feet will a body fall in 21 seconds down an incline of 7 in 16?

22. In a compound lever the arms of the power are 9, 7, 5, and 3 feet, the arms of the weight 3, 2, 1, and $\frac{1}{2}$ feet, the power is 11 lbs.; required the weight.

23. If mercury and milk are placed together in a bent glass tube or syphon, and if the column of mercury is 7·9 inches in length, what will be the length of the column of milk?

24. In a hydrostatic press the sectional areas of the cylinders are to one another as 1111 to 2, the force pump is worked by means of a lever whose arms are to one another as 17 to 2, and the power applied is 123 lbs.; what is the upward pressure exerted against the piston in the large cylinder?

25. In a differential screw the pitch of the exterior screw is $\frac{2}{7}$ of an inch, that of the interior screw is $\frac{2}{11}$ of an inch, the lever is 25 inches long and the power applied is 130 lbs., what is the pressure exerted?

26. A seconds pendulum is observed when carried to the summit of a mountain to lose 3 seconds in an hour; what is the height of the mountain?

27. Through how many feet will a heavy body fall during the 10th, the 7th and the 6th, seconds of its descent?

28. In what time will an upright vessel 20 feet high and filled with water, empty itself through an aperture, in the bottom, three-fifths of an inch in area, the vessel containing 250 gallons?

29. A train weighing 80 tons is drawn along a level plane with a uniform velocity of 20 miles per hour, taking friction and atmospheric resistance as usual, what is the horse power of the locomotive?

30. What is the weight of the milk contained in a rectangular vat 11 feet long, 7 feet wide, and 3 feet deep?

31. What would be the height of the mercury in the barometer at an elevation of 29·7 miles?

32. What power will support a weight of 666666 by means of an endless screw having a winch 30 inches long, an axle with a radius of 2 inches, and a wheel with 120 teeth?

33. How much work is required to raise 29 tons of coal from a mine 1120 feet deep?

34. With what velocity does a body move at the close of the 27th second of its descent?

35. What is the entire pressure exerted upon the body of a fish having a surface of 11 square yards and being at a depth of 140 feet?

36. How much water will be discharged in one hour through an aperture in the side or bottom of a vessel, the water in the vessel being kept at a constant height of 17 feet above the centre of the orifice, and the area of the latter being seven-elevenths of an inch?

37. How many cubic feet of water can a man raise by means of a chain pump from a depth of 120 feet in a day of 8 hours?

38. If a stone be thrown down an incline of 11 in 100 with an initial velocity of 140 feet per second, what will be its velocity at the 10th second of its descent, and through how many feet will it fall in 21 seconds?

39. At what rate per hour will a train weighing 120 tons be drawn up an incline of $\frac{1}{2}$ in 100 by an engine of 90 horse-power, taking friction as usual and neglecting atmospheric resistance?

40. A water-wheel is driven by a stream 4 feet wide and 3 feet deep, the fall is 40 feet and the velocity of the stream $38\frac{1}{2}$ feet per minute—if the modulus of the wheel is ·63, what number of gallons of water will it raise per hour from a depth of 270 feet?

41. In a system of movable pulleys a power of 2 lbs. sustains a weight of 64 lbs.; how many movable pulleys are there?
   1st. If the system be worked by one cord?
   2nd. If there are as many cords as movable pulleys?

42. At what rate per hour will a horse draw a load whose gross weight is 1800 lbs. on a road whose coefficient of friction is $\frac{1}{25}$?

43. In a high pressure engine the piston has an area of 600 inches and makes 30 strokes of 5 feet each per minute; what must be the pressnre of the steam on the boiler in order that the engine may pump 1000 gallons of water per minute from a mine whose depth is 270 feet—making the usual allowance for friction and the modulus of the pump?

44. The barometer at the summit of a mountain indicates a pressure of 21·73 inches while at the base it indicates a pressure of 29·44 inches, what is the height of the mountain, taking the mean temperature of the two stations as 63·70?

45. If a stone be thrown vertically upwards and again reaches the ground after a lapse of 16 seconds, to what height did it rise?

46. In a composition of levers the arms of the power are 8, 4, 2, and 7, the arms of the weight are 3, 1, $\frac{1}{2}$, and 4; what weight will be sustained by a power of 29lbs.?

47. A piece of wood which weighs 17 oz. has attached to it a metal sinker which weighs 13·7 oz. in air and 8·6 oz in water—the united mass weighs only ·5 of an ounce in water; what is the specific gravity of the wood?

48. What must be the area of an aperture in the bottom of a vessel of water 18 feet deep and kept constantly full in order that 27 cubic feet may be discharged per hour?

49. How many tons of coal will be raised per day of ten hours from a mine whose depth is 400 feet, by a low pressure engine in which the piston has an area of 1200 inches and makes 20 strokes of 6 feet each per minute, the pressure of the steam on the boiler being 45 lbs. to the sq. inch?

50. What power will support a weight of 70000 by means of a screw having a pitch of $\frac{7}{25}$ of an inch and a power lever 9 feet two inches in length?

51. In what time will a pendulum 50 inches long vibrate in the latitude of Canada?

52. In a lever whose power arm is $8\frac{1}{2}$ times as long as the arm of the weight, what power will sustain a weight of 729 lbs.?

53. A train weighing 130 tons is drawn along an incline of $\frac{1}{6}$ in 100 with a uniform velocity of 25 miles per hour; taking friction and atmospheric resistance as usual, what is the horse power of the locomotive:—

 1st. If the train is ascending the incline?
 2nd. If the train is descending the incline?

54. A seconds pendulum is observed to lose 40 seconds in 24 hours on the summit of a mountain; required its height.

55. A body is fired vertically with an initial velocity of 2000 feet per second; it is required to find:

 1st. Where it will be at the end of the 120th second.
 2nd. How far it will rise.
 3rd. In what space of time it will again reach the ground.
 4th. Its terminal velocity.
 5th. In what other moment of its flight its velocity will be the same as at the end of the 49th second.

56. In a wheel and axle the radius of the axle is 3 inches and a weight of 247 lbs. is sustained by a power of 17 lbs.; what is the radius of the wheel?

57. With what velocity does water flow from a small aperture in the side or bottom of a vessel, the fluid level being kept constant at 40 feet above the centre of the orifice?

58. In a train of wheel work there are four wheels and four axles, the first wheel and last axle being plain, i. e., without cogs, and having radii respectively of 12 and 2 feet—the second wheel has 70, the third 80, and the fourth 100 teeth: the first axle has 8, the second 7, and the third 11 leaves; with this machine what weight will be sustained by a power of 130 lbs.?

59. To what depth may a closed empty glass vessel capable of sustaining a pressure of 200 lbs. to the square inch be sunk in water before it breaks?

60. In a differential wheel and axle the radii of the axles are $1\frac{2}{5}$ and $1\frac{2}{9}$ inches: a power of 2 lbs. sustains a weight of 749, what is the radius of the wheel?

61. How many units of work are expended in raising 247 tons of coal from a depth of 478 feet?

62. What is the horse power of an upright water wheel worked by a stream 5 feet wide and $2\frac{1}{2}$ feet deep, the velocity of the water being 110 feet per minute, the fall 6 feet, and the modulus of the wheel $\frac{2}{5}$?

63. A train weighing 140 tons ascends a gradient having a rise of $\frac{1}{2}$ in 100; taking friction as usual and neglecting atmospheric resistance, what is the maximum speed the train will attain if the horse power of the locomotive be 150?

64. A barometer at the summit of a mountain indicates a pressure of 21·4 inches, while at the base the pressure is 30·2 inches, what is the height of the mountain?

65. On an incline of 7 in 100 what power acting parallel to the plane will sustain a weight of 947 lbs.?

66. What centrifugal force is exerted by a ball weighing 40 lbs, revolving in a circle 20 feet in diameter and making 140 revolutions per minute?

67. What is the specific gravity of a piece of metal which weighs 23·49 oz. in air and only 18·12 oz. in water?

68. If a body be thrown vertically upwards and again reaches the ground in 22 seconds—
    1st. With what velocity was it projected?
    2nd. How far did it rise?

69. In a screw the pitch is $\frac{4}{19}$ of an inch, the power lever is 40 inches long; what power will sustain a weight of 95000 lbs?

70. In what time will an engine of 120 horse power, moving a train whose gross weight is 100 tons, complete a journey of 300 miles, taking friction as usual, neglecting atmospheric resistance, and assuming the rail to ascend regularly $\frac{1}{8}$ in 100?

71. An engine of 60 horse power raises 50 tons of coal per hour from the bottom of a mine 200 feet deep, and at the same time causes a forge hammer to make forty lifts per minute of 3 feet each; required the weight of the hammer.

72. In a hydrostatic press the sectional areas of the cylinders are to one another as 1411 to 3, the force pump is worked by a lever whose arms are to one another as 28 to 3, the upward pressure required is 9000 lbs; what must be the power applied?

73. In a differential screw the pitch of the exterior screw is $\frac{3}{19}$ and that of the inner screw $\frac{4}{17}$ of an inch, the power lever is 6 feet 8 inches in length; what pressure will be exerted by a power of 19 lbs.?

74. A piece of nickel (spec. grav. 7·816) weighs 24 grains in air and only 16·4 grains in a certain liquid; required the specific gravity of the liquid.

75. In a differential wheel and axle the radii of the axles are $1\frac{1}{4}$ and $1\frac{3}{19}$ inches, the radius of the wheel is 42 inches; what weight may be sustained by a power of 23·7 lbs.?

76. What gross load will a horse draw when travelling at the rate of $3\frac{1}{2}$ miles per hour on a road whose coefficient of friction is $\frac{1}{16}$?

77. A body has descended through $a + x$ feet when a second body commences to fall at a point $2m$ feet beneath it; what distance will the latter body fall before the former passes it?

78. On an incline of $\frac{1}{2}$ in 70 what power acting parallel to the plane will sustain a weight of 4790 lbs.?

79. When a body has fallen 7000 feet down an incline of 7 in 20 what velocity per second has it acquired?

80. A body weighing 100 lbs. and moving from south to north with a velocity of 60 feet per second comes into contact with another body which weighs 430 lbs. and is moving from north to south with a velocity of 20 feet per second, and the two bodies coalesce and move on together; required the direction and velocity of the motion of the united mass.

81. An engine of 21 horse power pumps 40 cubic feet of water per hour from the bottom of a mine whose depth is 200 feet and at the same time draws coals from the bottom of the mine; required the tons of coals drawn up per hour.

82. In a system of pulleys worked by several cords, each attached by both ends to the pulleys, there are 8 movable pulleys and as many separate cords; what weight will be sustained by a power of 73 lbs.?

83. A body weighing 20 lbs. and moving at the rate of 47 feet per second comes in contact with another body weighing 270 lbs. and moving in the same direction with the velocity of 30 feet per second; required the velocity and momentum of the united mass.

84. In what time will an engine of 150 horse power draw a train whose gross weight is 90 tons through a journey of 220 miles, taking friction as usual, and neglecting atmospheric resistance, one half of the journey to be on a level plane and the other half up an incline of $\frac{3}{4}$ in 100?

85. In a common wheel and axle a power of 7 lbs. sustains a weight of 974; the radius of the wheel is 51 inches, what is the radius of the axle?

86. At what height would the mercury stand in a barometer placed at an elevation of 43·2 miles above the level of the earth?

87. If a body be projected down an incline of 7 in 12 with an initial velocity of 40 feet per second, through how many feet will it move during the tenth second, and over what space will it have passed in 23 seconds?

88. In a high pressure engine the piston has an area of 360 inches and makes 17 strokes of 5 feet each per minute; taking the pressure of the steam on the boiler as equal to 56 lbs. to the square inch, what are the horse powers of the engine?

89. If a body weighing 111 lbs. moving to the east with a velocity of 90 feet per second come in contact with another body which weighs 70 lbs. and is moving to the west with a velocity of 40 feet per second, and after the two have coalesced they come in contact with a third which weighs 80 lbs. and is moving to the east with a velocity of 20 feet per second, and the three then coalesce and move on together, what will be the direction, velocity, and momentum of the united mass?

90. What must be the length of a pendulum in the latitude of Canada in order that it may make 40 vibrations in 1 minute?

91. What pressure will be exerted upon the body of a man at the depth of 97 feet beneath the surface of the water, the man's body having a surface equal to 14 square feet?

92. A piece of cork which weighs 27·42 grains has attached to it a sinker which weighs 34·71 grains in air and only 30·12 grains in water, the united mass weighs nothing in water, i. e., is of the same specific gravity as water; required the specific gravity of the cork.

93. What is the weight of a mass of slate which contains 27 cubic feet?

94. How many cubic feet of iron are there in 87 tons?

95. What backward pressure is exerted by a horse in going down a hill which has a rise of 3 in 40 with a load whose gross weight is 2100 lbs. assuming friction to be equal to $\frac{1}{30}$ of the load?

96. What pressure is exerted against one square yard of an embankment if the upper edge of the yard be 17 feet and the lower edge 18 feet beneath the surface of the water?

97. The length of a wedge is 27 inches, and the thickness of the back $2\frac{3}{8}$ inches; what weight may be raised by a pressure of 17 lbs.?

98. What is the effective horse power of a high pressure engine in which the piston has an area of 420 inches, and makes 30 strokes per minute, the boiler evaporating $\frac{1}{6}$ of a cubic foot of water per minute under a pressure of 60 lbs. to the square inch?

99. A train drawn by a locomotive of 100 H. P. moves along an incline of $\frac{1}{4}$ in 100 with a uniform velocity of 25 miles per hour; taking friction as usual and neglecting atmospheric resistance, what is the weight of the train:—
   1st. If it is ascending the incline?
   2nd. If it is descending the incline?

100. A lightning flash is seen $9\frac{3}{4}$ seconds before the report is heard, at what distance did the discharge occur?

101. A body 7000 miles from the surface of the earth weighs 500 lbs., what would be its weight at the distance of 4000 miles?

102. How long would sound require to travel from Toronto to Markham, a distance of 21 miles, the thermometer indicating a temperature of 82° F.?

103. At what distance from the source of sound must the reflecting surface be in order that the last 20 syllables uttered may be distinctly repeated by the echo?

104. On a perfectly calm day the report of a cannon fired on the northern shore of Lake Ontario is heard on the southern shore, a distance of 40 miles. How much sooner will the report arrive at the southern shore through the water of the lake than through the overlying air?

105. A metallic wire placed on the monochord vibrates 800 times in a second—by how much must its length be increased in order that with the same degree of tension, &c., it shall vibrate only 550 times in a second?

106. What are the relative numbers of vibrations per second required to produce the notes E and D sharp?

107. What is the length of a wave of air producing F^2 of the Italian Opera (1855)?

108. A cord of certain length and diameter makes 90 vibrations per second when stretched over the sonometer by a weight of 100 lbs., by what weight must it be stretched in order to make 135 vibrations per second?

109. In the year 1783, the report of a meteor was heard at Windsor Castle 10 minutes after the flash of the meteor was seen, what was its distance assuming the temperature of the air to be 52° F.?

110. An upright vessel is filled with water and is pierced in the side at the heights of 2, 5, 9, and 16 feet from the ground, taking the whole height of the water as 24 feet, what in each case will be the random of the jet?

111. A person supposes himself to be in the range of a distant cannon, the report of which he hears 23 seconds after seeing the flash, how soon may he apprehend danger from the ball assuming that it travels with the uniform velocity of $\frac{1}{10}$ of a mile per second?

EXAMINATION PAPERS.

I.

1. A railway train weighing 110 tons is drawn along an incline of $\frac{1}{4}$ in 100 with a uniform velocity of 42 miles per hour, taking friction as usual and atmospheric resistance equal to 20 lbs. when the train is moving at the rate of 7 miles per hour, what is the horse power of the locomotive?
   1st. If the train is ascending the gradient?
   2nd. If the train is descending the gradient?
2. Enunciate the principle of virtual velocities and calculate through how many feet a weight of 89·7 lbs. will be carried by a power of 11·7 lbs. moving through 123 feet?
3. In a differential wheel and axle the radii of the axles are $3\frac{1}{7}$ and $3\frac{1}{9}$ inches; the radius of the wheel is 42 inches, what power will sustain a weight of 444·4 lbs.?
4. Describe the barometer, and explain the principles on which it acts.
5. What is the weight of a log of boxwood (spec. grav. 1·320) 17 feet long, 1 foot 9 inches wide, and 2 feet 3 inches thick?
6. The upright gate of a canal is 12 feet wide and 16 feet deep, the water being on one side only and level with the top; required the pressure :—
   1st. On the whole gate.
   2nd. On the lowest five-eighths of the gate; and,
   3rd. On the middle seventh of the gate.
7. Give the composition of atmospheric air, and state what are the chief sources of the carbonic acid.
8. The piston of a high pressure engine has an area of 400 inches, and makes 12 strokes of 6 feet each per minute, the pressure of the steam on the boiler is 64 lbs. per square inch; how many tons of coal per hour will this engine raise from a mine whose depth is 240 feet?

9. Distinguish between the essential and the accessory properties of matter, and enumerate the former.

10. An upright vessel 17 feet in height is filled with water and holds just 200 gallons; in what time will it empty itself through an aperture in the bottom two-fifths of an inch in area?

II.

1. A cannon ball is fired vertically with an initial velocity of 800 feet per second; required—
   1st. How far it will ascend.
   2nd. In what space of time it will again reach the ground.
   3rd. Where it will be at the end of the 31st second.
   4th. Its terminal velocity.
   5th. In what other moment of its flight it will have the same velocity as at the close of the 13th second.

2. Enumerate the different kinds of attraction, define what is meant by the attraction of gravity, and state by what law its intensity varies.

3. A piece of stone weighs 73 grains in air and only 35 grains in water; required its specific gravity.

4. In a hydrostatic press the areas of the cylinders are to one another as 1347 : 2, the force pump is worked by means of a lever whose arms are to one another as 23 : 2, the power applied is 120 lbs.; required the upward pressure exerted against the piston in the larger cylinder.

5. In a lever the power arm is 7 feet long, the arm of the weight is 5 inches, the power is 11 lbs.; required the weight.

6. Enunciate the principle of the parallelogram of forces, and explain how it is that a force may be more advantageously represented by a line of given length than by saying it is equal to a given number of lbs., &c.

7. Name the different kinds of upright water wheels, explain the difference between them, and give the rule for finding the horse powers of a water wheel.

8. If a closed empty vessel be sunk in water to the depth of 143 feet before it breaks, what was the extreme pressure to the square inch it was capable of sustaining?

9. Describe what is meant by the *vena contracta* of escaping fluids, indicate its position with reference to the orifice of escape, and give the proportion between the area of the aperture and the sectional area of the *vena contracta*.

10. An engine of 50 horse power draws a train weighing 60 tons up an incline of ¼ in 100 with a uniform velocity of 20 miles per hour; taking atmospheric resistance as usual, what is the friction per ton?

III.

1. By means of a lever a certain number of lbs. Troy attached to the arm of the weight balances the same number of ounces Avoirdupois attached to the arm of the power; required the ratio between the arms of the lever, a pound Troy being to a pound Avoirdupois as 5760 : 7000.
2. Enunciate *Torricelli's theorem* and calculate the velocity with which a liquid spouts from a small orifice in the side of a vessel when the level of the fluid is 24 feet above the centre of the orifice.
3. In a hydrostatic bellows the sectional area of the tube is three-sevenths of an inch and it contains 12 lbs. of water, the area of the board of the bellows is 3·7 square feet; what is the upward pressure exerted against the board of the bellows?
4. Through how many feet will a body fall during the 22nd second of its descent?
5. Define what is meant by *specific gravity*. Give the rule for calculating the specific gravity of a solid not sufficiently heavy to sink in water, and calculate the specific gravity of cork from the following data:—

   A piece of cork which weighs 20 oz. in air has attached to it an iron sinker which weighs 18 oz. in air and only 15.73 oz. in water; the united mass weighs 1 oz. in water, required the specific gravity of the cork.
6. What weight would be carried through a space of 7 feet by a power of 5 lbs. moving through 40 feet?
7. Define what is meant by the *centre of gravity* of a body, and explain how it may be experimentally determined in a solid.
8. How many tons of coal per day of ten hours may be raised from a mine of 660 feet in depth by a low pressure engine having a piston which has an area of 500 inches, and makes 20 strokes of 11 feet each per minute, the gross pressure of the steam on the boiler being 37 lbs. per square inch?
9. The power arm of a lever is 32 times as long as the arm of the weight, the power is 97 oz.; required the weight.

10. A city is supplied with water through a pipe 8 inches in diameter and 1 mile in length, leading to a reservoir whose height is 140 feet above the remote end of the pipe; what will be the velocity of the water per second, and how much will be discharged in one hour?

IV.

1. Enunciate the law of decrease in the pressure and density of the air as we ascend into the higher regions of the atmosphere?

2. In a hydrostatic press the sectional areas of the cylinders are to one another as 943 : 2, the force pump is worked by means of a lever whose arms are to one another as 19 : 3; if the power applied be 87 lbs., what is the upward pressure exerted against the piston in the larger cylinder?

3. The power arm of a lever is 9 feet long, the arm of the weight is 17 feet long, and the weight is 6½ lbs.; required the power.

4. Explain when a body is said to be in a condition of *stable*, *unstable* or *indifferent equilibrium*.

5. A train weighing 90 tons is drawn along an incline of 2 in 900 with a uniform velocity of 30 miles per hour; taking friction and atmospheric resistance as usual, what is the horse power of the locomotive:—
 1st. If the train is ascending the gradient?
 2nd. If the train is descending the gradient?

6. A stone is dropt into a mine and is *heard* to strike the bottom in 11½ seconds; required the depth of the mine, if sound travels at the rate of 1066⅔ feet per second.

7. State the condition of equilibrium in the differential wheel and axle.

8. What is the weight of the sulphuric acid (specific gravity 1·841) contained in a rectangular vat 7 feet 4 inches long, 2 feet 5 inches deep, and 3 feet 7 inches wide?

9. At the top of a mountain a barometer indicates a pressure of 21 inches while at the base the pressure is 29·78 inches—the temperature at the top is 40·70° Fahr. and that at the base is 70·70° Fahr.; required the height of the mountain.

10. A high pressure steam engine raises 200 cubic feet of water per minute from a depth of 80 feet, the piston has an area of 800 inches, and makes 10 strokes per minute of 8 feet each, what is the pressure of the steam on the boiler?

V.

1. The flood gate of a canal is 10 feet long and 7 feet deep, the water being on one side and level with the top; what is the pressure :—
   1st. On the whole gate?
   2nd. On the lowest two-sevenths of the gate?
   3rd. On the middle three-sevenths of the gate?
   4th. On the lowest one-ninth of the gate?
2. In a compound lever the arms of the power are 6, 7, and 11 feet, the arms of the weight are 2, 3, and 5 feet; by means of this combination what power will sustain a weight of 1000 lbs.?
3. Enunciate Mariotte's law, and ascertain what will be the density, volume and elasticity of that amount of atmospheric air, which, under ordinary circumstances, i. e., at the level of the sea or under a pressure of 15 lbs. to the square inch, fills a gallon measure, if it be placed in a piston and subjected to a pressure of 60 lbs. to the square inch.
4. What power moving through 29 feet will carry a weight of 7 lbs. through 70 feet?
5. An engine of 12 horse power gives motion to a forge hammer which weighs 400 lbs. and makes 40 lifts of 3 feet each per minute and at the same time pumps water from a mine 100 feet deep; required the number of cubic feet of water it pumps per hour from the mine.
6. On an inclined plane a power of 341 lbs. acting parallel to the base sustains a weight of 27,900 lbs.; what must be the length of the base in order that the height may be 11 feet?
7. Enunciate the three laws of motion commonly known as Newton's laws, and state to whom they respectively belong.
8. A piece of sulphur weighs 19 oz. in air and 10 oz. in water, required its specific gravity.
9. A ball is thrown up an incline of 11 in 16 with an initial velocity of 1100 feet per second; required—
   1st. To what height it will rise.
   2nd. Where it will be at the end of the 20th second.
   3rd. In what time it will again reach the ground.
   4th. Its terminal velocity.
   5th. In what other moment of its flight it will have the same velocity as at the end of the 17th second of its ascent.
10. Required the pressure exerted against a mill-dam 170 feet long and 16 feet wide, the perpendicular depth of the water being 12 feet.

VI.

1. When the barometer indicates a pressure of 30 inches at the surface of the earth it is observed to indicate a pressure of of only 13·5 inches in a balloon, required the approximate height of the balloon.
2. Give the chief laws connected with the motion of projectiles.
   1st. When they are fired vertically, and
   2nd. When they are fired at an angle of elevation.
3. Through how many feet will a body fall in 30 seconds?
4. What is the horse power of a low pressure engine in which the piston has an area of 360 inches and makes 11 strokes of 9 feet each per minute, the gross pressure of the steam on the boiler being 53 lbs. to the square inch?
5. What must be the area of the aperture in the side of a vessel kept filled with water to a height of 20 feet above the centre of the orifice in order that 15 cubic feet of water may be discharged in one hour?
6. Describe Bramah's Hydrostatic Press, and explain upon what principle in philosophy its action depends.
7. A piece of wood which weighs 19 oz. has attached to it a metal sinker which weighs 27 oz. in air and 22·7 oz. in water—the united mass weighs 11 oz. in water; required the specific gravity of the wood.
8. In a compound lever the arms of the power are 7, 8, 9, and 10 feet, the arms of the weight are 2, 3, 4, and 1 feet, the power is 19 lbs.; what is the weight?
9. Explain the difference between the common and the forcing pump, and also state why the former is sometimes called the *lifting pump*.
10. A train weighing 80 tons is moving at the rate of 30 feet per second when the steam is turned off, how far will it ascend an incline of 3 in 1000, taking friction as usual, and neglecting atmospheric resistance?

VII.

1. What amount of pressure is exerted against one square yard of an embankment, the upper edge of the square yard being 16 ft. 3 in. and the lower edge 19 ft. 9 in. below the surface of the water?
2. How much must the pendulum of a clock which loses 1 minute in an hour be shortened in order that it may keep correct time?

3. Describe the syphon and give the theory of its action.

4. In a system of eleven movable pulleys worked by a single cord what weight will a power of 27 lbs. sustain ?

5. In a hydrostatic press the large cylinder has a sectional area of $6\frac{1}{2}$ feet, the smaller cylinder a sectional area of $2\frac{1}{4}$ inches, the force pump is worked by means of a lever whose arms are to one another as 19 : $1\frac{1}{2}$; now if a power of 100 lbs. be applied to the extremity of the lever, what upward pressure will be exerted against the piston in the larger cylinder ?

6. Describe the differential screw, and give the conditions of equilibrium between the power and weight in the common screw.

7. To what depth may an empty glass vessel capable of sustaining a pressure of 197 lbs. to the square inch be sunk in water before it breaks ?

8. In a system of pulleys consisting of eight movable pulleys worked by eight cords, the upper end of each fastened to the beam the power is $7\frac{1}{2}$ lbs., what is the weight ?

9. How many gallons of water per hour will an engine of 7 horse power pump from a mine 67 feet in depth, making the usual allowance for the modulus of the pump ?

10. The piston of a low pressure engine nas an area of 400 inches and makes 20 strokes, each eight feet in length, per minute, the boiler evaporates ·731 of a cubic foot of water per minute, what is the useful horse power of the engine ?

VIII.

1. Explain the difference between the simple and compound pendulum — also what is meant by the " centre of oscillation" and by the "centre of percussion."

2. What velocity will a heavy body falling freely in the latitude of London acquire in one entire second, the London seconds pendulum being 39·13 inches long ?

3. In a hydrostatic bellows the tube is filled with water to the height of $13\frac{1}{2}$ feet; what upward pressure is exerted against the board of the bellows if the area of the latter be $3\frac{7}{12}$ feet ?

4. In a differential screw the exterior screw has a pitch of $\frac{4}{17}$ of an inch, the interior screw a pitch of $\frac{5}{21}$ of an inch, the power lever is fifty inches long ; what pressure will be exerted by a power of 130 lbs. ?

5. A train weighing 100 tons moves up a gradient having an inclination of $\frac{5}{8}$ in 100 with a uniform speed of 20 miles per hour; taking friction and atmospheric resistance as usual, what is the horse power of the locomotive?

6. When a body has fallen through 2500 feet what velocity has it acquired?

7. Explain what is meant by gaseous diffusion, and show the important influence it has in maintaining the composition of atmospheric air constant at all places.

8. In a common wheel and axle the radius of the axle is 11 inches and the radius of the wheel 47 in.; what power will, with this machine, sustain a weight of 793 lbs.?

9. A flood gate is 22 feet wide and 20 feet deep, the water being on one side only and level with the top; required the pressure—

   1st. Against the whole gate.

   2nd. Against the lowest three-sevenths.

   3rd. Against the upper four-ninths.

   4th. Against the middle three-elevenths.

   5th. Against the lowest three-fifths.

10. Give the different rules for finding the specific gravity of liquids.

IX.

1. In a differential wheel and axle the radii of the axles are $2\frac{3}{7}$ and $2\frac{3}{11}$ inches, the radius of the wheel is 90 inches; what weight will be sustained by a power of 7 lbs.?

2 The tube of a hydrostactic bellows is filled with water to the height of 50 feet; if the board of the bellows has an area of $6\frac{7}{8}$ feet, what upward pressure is exerted against it?

3. How many vibrations per minute will a pendulum 9 yards long make?

4 Give the principal laws of the descent of bodies on inclined planes.

5. A body has fallen through 3600 feet when another body begins to fall at a point 4000 feet beneath it; through what space will the latter body fall before the first overtakes it?

6. The piston of a steam engine has an area of 440 inches and makes 11 strokes per minute, each $9\frac{1}{11}$ feet in length, the boiler evaporates ·9 of a cubic foot of water per minute; what is the volume of the steam produced per minute and what is the pressure under which it is generated?

7. Give the most important consequences that result from the fact that each atom of a liquid is separately drawn towards the centre of the earth by the force of gravity.
8. What gross load will a horse exerting a traction of 74 lbs. draw on a road whose coefficient of friction is $\frac{1}{24}$?
9. What are the conditions of equilibrium between the power and weight in the inclined plane?
10. Through how many feet must a body fall in order to acquire a velocity of 250 feet per second?

ANSWERS AND REFERENCES TO EXAMINATION PAPERS.

I

1. H. P.=228·48 or 105·28
2. Art. 66.
3. ·1679 lbs.
4. Arts. 227, 229.
5. 5522·34375 lbs.
6. 96000 lbs., 82500 lbs., and 13714$\frac{2}{7}$ lbs.
7. Art. 205.
8. 151·2 tons.
9. Arts. 9, 19, and 20,
10. 18 min., 45 sec.

II.

1. H. P.=161·28 or 38·08.
2. Arts. 25, 27.
3. 1·921.
4. 929430 lbs.
5. 184$\frac{4}{5}$ lbs.
6. Art. 44.
7. Arts. 339, 341.
8. 62·05 lbs. 341.
9. Arts. 9, 19, and 10.
10. 8·425 lbs. per ton.

III.

1. Power arm 13$\frac{29}{175}$ times as great as the arm of the weight.
2. Arts. 25, 26, and 27.
3. 14918·4 lbs.
4. 688 feet.
5. Arts. 192, 195, and ·57584.
6. 28$\frac{4}{7}$ lbs.
7. Arts. 57, 58.
8. 1400.
9. 194 lbs.
10. Velocity= 6·336 feet per second.
    Quantity—7962·071 cubic feet per hour.

IV.

1. Art. 212.
2. 259796$\frac{1}{2}$ lbs.
3. 12$\frac{5}{18}$.
4. Art. 62.
5. H. P. = 106 16 or 42·16.
6. 1600 feet.
7. Art. 88.
8. 7307·00144 lbs.
9. 9721·2 feet.
10. 33$\frac{6}{7}$ to the square inch.

V.

1. $15312\frac{1}{2}$ lbs., 7500 lbs., $6562\frac{1}{2}$ lbs., and $3213\frac{19}{162}$ lbs.
2. $64\frac{72}{77}$ lbs.
3. Art. 219, density 4 times as great, volume 1 qt. and elasticity 60 lbs. to the sq. inch.
4. $16\frac{28}{29}$ lbs.
5. $3340\frac{4}{5}$ cubic feet.
6. 900 feet.
7. Arts. 255, 256, 257.
8. 2·111.
9. 27500 feet.
   At elevation of 17600 feet.
   100 seconds.
   1100 feet per second.
   At the end of the 83rd sec.
10. 1020000 lbs.

VI.

1. 19724 feet.
2. Art. 282.
3. 24336 feet.
4. 45·36 H. P.
5. $\frac{15}{559}$ of an inch.
6. Arts. 183 and 182, Note.
7. ·618.
8. 3990 lbs.
9. Arts. 233 and 234.
10. 2163·4 feet.

VII.

1. $10406\frac{1}{4}$ lbs.
2. 1·303 inches.
3. Art. 235.
4. 594 lbs.
5. $526933\frac{1}{3}$ lbs.
6. Arts. 129, 126.
7. 454 feet.
8. 1920 lbs.
9. $13791\frac{3}{67}$ gallons.
10. H. P. = 67·87.

VIII.

1. Arts. 301, 302, and 308.
2. 386·17 inches.
3. 3022·68672.
4. 14660165·6 lbs.
5. 133·262.
6. 400 feet per second.
7. Art. 206.
8. $185\frac{28}{47}$ lbs.
9. 275000 lbs.
   $185204\frac{4}{49}$ lbs.
   $54320\frac{80}{81}$ lbs.
   75000 lbs.
   231000 lbs.
10. Art. 196.

IX.

1. 8085 lbs.
2. 21479·04 lbs.
3. 20·8.
4. Art. 270.
5. $1111\frac{1}{9}$ feet.
6. Volume = 339·5 cub. feet.
   Pres. = 85 lbs. the sq. inch.
7. Art. 175.
8. 1776 lbs.
9. Art. 116.
10. $976\frac{9}{10}$ feet.

QUESTIONS TO BE ANSWERED ORALLY BY THE PUPIL.

NOTE.—*The numbers following the questions refer to the numbered articles in the work, where the answers may be found.*

1. What is Natural Science? (1)
2. Into what classes are all natural objects divided, and how are these distinguished from each other? (2)
3. How are animals distinguished from vegetables? (3)
4. What is Zoology? (4)
5. What is Botany? (4)
6. What is Mineralogy? (4)
7. What is Astronomy? (4)
8. What is Geology? (4)
9. What is Chemistry? (4)
10. What is the object of Natural Philosophy? (4)
11. What are the subdivisions of Natural Philosophy? (5)
12. In what separate forms does matter exist? (6)
13. Define what is meant by the *essential* properties of matter. (9)
14. Enumerate the essential properties of matter. (10)
15. What is extension? (11)
16. What is impenetrability? Give some illustrations. (12)
17. What is divisibility? (13)
18. Does the property of divisibility belong to masses or to particles of matter or to both? (13)
19. Give some illustrations of the extreme divisibility of matter. (13, Note)
20. What is indestructibility? (14)
21. What is Porosity? (15)
22. What is Compressibility? (16)
23. What is Inertia? (17)
24. If bodies cannot bring themselves to a state of rest, how is it that all bodies moving upon or near the earth soon come to rest? (17, Note)
25. What is elasticity? (18)
26. Name the different kinds of elasticity as applied to solids. (18, Note)

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27. What are the *accessory* properties of matter? (19)
28. Enumerate some of the most important of the accessory properties of matter? (20)
29. What is malleability? Which are the most malleable of metals? (21)
30. What is ductility? Name the most ductile metals. (22)
31. What is tenacity? (23)
32. What is attraction? (24)
33. Enumerate the different kinds of attraction. (25)
34. What is the attraction of gravity? (26)
35. What is the law of variation in the intensity of gravity? (27)
36. Explain what is meant by saying the force of gravity varies inversely as the square of the distance. (28)
37. What is the attraction of cohesion? (29)
38. What is the attraction of adhesion? (30)
39. What is capillary attraction? Give some examples. (31)
40. What is electrical attraction? (32)
41. What is magnetic attraction? (33)
42. What is chemical attraction? (34)
43. What is the derivation of the word *Statics?* (36)
44. What is the object of the science of Statics? (36)
45. What is the derivation of the word *Hydrostatics?* (36)
46. What is the object of the science of Hydrostatics? (36)
47. What is the derivation of the word *Dynamics?* (36)
48. What is the object of the science of Dynamics? (36)
49. What is the derivation of the word *Hydrodynamics?* (36)
50. What is the object of the science of Hydrodynamics? (36)
51. What is the derivation of the word *Pneumatics?* (36)
52. What is the object of the science of Pneumatics? (36)

53. When is a body said to be in equilibrium? (37)
54. What are statical forces or pressures? (38)
55. What are the elements of a force? (39)
56. What are the different modes of representing a force? (40)
57. When several forces act upon the same point of a body, how many motions can they give it? (41)
58. Distinguish between *component* and *resultant forces.* (42)
59. If several forces act upon a point in the same straight line and in the same direction, to what is their resultant equal? (43)
60 When several forces act upon a point in the same straight line, but in opposite directions, to what is their resultant equal? (43)
61. Enunciate the principle of the parallelogram of forces. (44)
62. When several forces act on a point in any direction whatever, state how the resultant may be found. (45)
63. What is the distinction between the parallelogram of forces and the parallelopiped of forces? (46)
64. What is the resultant of two parallel forces which act on different points of a body, but in the same direction? (47)
65. What is the resultant of two parallel forces which act on different points of a body and in opposite directions? (48)
66. How do we find the resultant of any number of parallel forces? (49)
67. What is a *couple?* (50)
68. Distinguish between the composition of forces and the resolution of forces. (54)
69. What is the centre of gravity of a body? (57)
70. Why is the centre of gravity called also the centre of parallel forces? (55)
71. How may the centre of gravity of a solid body be experimentally determined? (58)
72 If a body be free to move in any direction, how will it finally rest with reference to its centre of gravity? (60)
73. How is the stability of a body estimated? (61)
74. When is a body said to be in a condition of *stable, unstable,* or *indifferent* equilibrium? (62)
75. How may the centre of gravity of two separate bodies be found? (63)

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76. What is the object of all mechanical contrivances? (64)
77. By what law or principle in philosophy is the relative gain or loss of power and velocity in a machine determined? (65)
78. Enunciate the principal of virtual velocities. (66)
79. What is a machine? (67)
80 How many mechanical elements enter into the composition of machinery? (68)
81. Name the primary mechanical elements. (68)
82. Name the secondary mechanical element. (68)
83. From what mechanical element is the wheel and axle formed? (69)
84. Of what mechanical element are the wedge and screw modifications? (69)
85. How are levers, cords, &c., regarded in theoretical mechanics? (70)
86. What is a lever? (71)
87. Of how many kinds are levers? (72)
88. Of simple straight levers how many kinds are there? (73)
89. Upon what does the distinction between the three kinds of levers depend? (73)
90. Give examples of levers of the first class. (74)
91. How are the fulcrum, power, and weight placed in levers of the first class? (75)
92. How are the fulcrum, power, and weight placed in levers of the second class? (75)
93. Give some examples of levers of the second class. (75)
94. How are the fulcrum, power, and weight placed in levers of the third class? (76)

95. Give some examples of levers of the third class. (76)
96. In levers of the first class which must be greatest, the power or the weight? (76, Note)
97. In levers of the second class which must be greatest, the power or the weight? (76, Note)
98. In levers of the third class which must be greatest, the power or the weight? (76, Note)
99. What is the arm of the weight? What is the arm of the power? (77)
100. What are the conditions of equilibrium between the power and the weight in the lever? (77)
101. Deduce formulas for finding the power, the weight, the arm of the power or the arm of the weight when the other three are given. (77)
102. When the arms of the lever are curved or bent, how must their effective lengths be determined? (79)
103. What is a compound lever or composition of levers? (80)
104. Deduce rules for finding the power or the weight in a compound lever. (81)

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105. Describe the wheel and axle. (82)
106. Why is the wheel and axle sometimes called a *perpetual* lever? (84)
107. What are the conditions of equilibrium in the wheel and axle? (85)
108. Deduce a set of rules for finding the power, the weight, the radius of the axle or the radius of the wheel when the other three are given. (86)
109. Describe the differential wheel and axle. (87)
110. To what is it, in effect, equivalent? (87)
111. Deduce a set of rules for the differential wheel and axle.
112. In toothed gear how is the ratio between the power and the weight determined? (89)
113. How are axles commonly made to act on wheels? (90)
114. When is wheel work used to concentrate power? Give an example. (92)
115. When is wheel work used to diffuse power? Give an example. (92)
116. What are the conditions of equilibrium in a system of toothed wheels and pinions? (93)
117. What is a *pinion?* what are *leaves?* (91)
118. Deduce formulas for finding the power and the weight, in a system of wheels and axles. (94)
119. Explain what is meant by the *hunting cog*. (95)
120. Name the different kinds of wheels. (96)
121. Explain the difference between *crown, spur*, and *bevelled* gear. (97)
122. Explain for what purpose crown, spur, or bevelled gear is used. (98)
123. When bevelled wheels of different diameters are to be used together show how the sections of the cones of which they are to be frusta are found. (99)

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124. What is a pulley? (100)
125. Show that from the pulley itself no mechanical advantage is derived. (101)
126. Wherein consists the real advantage of the pulley and cord as a mechanical power? (101)
127. When is a pulley said to be fixed? (102)
128. What is a single movable pulley called? (103)
129. What are Spanish Bartons? (103)
130. Explain the meaning of the words *sheef, block* and *tackle*. (104)
131. What is the only mechanical advantage derived from the use of a fixed pulley? (105)
132. In a system of pulleys worked by a single cord, what are the conditions of equilibrium? (106)
133. Deduce a set of rules for a system of pulleys worked by a single cord. (107)
134. What are the conditions of equilibrium in a Spanish Barton when the separate cords are attached directly to the beam? (108)

135. What are the conditions of equilibrium when the separate cords are attached to the movable pulleys? (109)
136. Deduce in each of these last two cases a set of rules for finding the ratio between the power and the weight. (110, and 111)
137. If the lines of direction of the power and weight make with one another an angle greater than 120°, what is the relation between the power and the weight? (112)
138. In theoretical mechanics how is the inclined plane regarded? (113)
139. What are the modes of indicating the inclination of the plane? (114)
140. In the inclined plane how may the power be applied? (115)
141. What are the conditions of equilibrium in the inclined plane? (116)
142. Deduce a set of rules for the inclined plane. (117)
143. What is the wedge? (118)
144. How is the wedge worked? (119)
145. What are the conditions of equilibrium in the wedge when it is worked by pressure? (120)
146. In what important particular does the wedge differ from all the other mechanical powers? (120, Note 1)
147. Give some examples of the application of the wedge to practical purposes. (120, Note 2)
148. Deduce a set of rules for the wedge. (121)

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149. Describe the screw. (122)
150. How is the screw related to an ordinary inclined plane? (122, Note)
151. What is the *pitch* of the screw? (123)
152. How is the screw commonly worked? (124 and 125)
153. What are the conditions of equilibrium in the screw? (126)
154. How may the efficiency of the screw as a mechanical power be increased? (127)
155. Deduce a set of rules for the common screw. (128)
156. By whom was the *differential* screw invented? (129)
157. Upon what principle does the differential screw act? (129)
158. To what is the differential screw in effect, equivalent? (129)
159. Deduce a set of rules for the differential crew. (130)
160. Describe the endless screw. (131)
161. What are the conditions of equilibrium in the endless screw? (132)
162. Deduce a set of rules for the endless screw. (133)
163. How does friction affect the relation between the power and the weight in the mechanical elements? (135)
164. What are the different kinds of friction? (136)
165. What is meant by the *coefficient* of friction? (137)
166. What is the coefficient of sliding friction? (138)
167. What is the coefficient of friction on railways? (138)
168. What is the coefficient of friction on good macadamized roads? (138)
169. What is meant by the force of traction? (138)
170. Enumerate the different expedients in common use for diminishing friction? (139)
171. Give Coulomb's conclusions as regards sliding friction. (139)
172. Give Coulomb's conclusions as regards rolling friction. (139)

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173. What is the *unit* of work? (140)
174. How are units of work expended in raising a body found? (141)
175. What are the most important sources of laboring forces? (142)
176. How many units of work are there in one horse power? (142)
177. What is meant by the Table in Art? 142
178. What is the true work of the horse per minute? (142, Note)
179. In moving a carriage along a horizontal plane, for what purpose is work expended?
180. In the case of railway trains what is the amount of friction? (143)
181. In the case of railway trains when does the velocity become uniform?

182. Upon what does the traction of force with which an animal pulls depend? (146)
183. At what rate per hour must a horse travel to do most work? (146)
184. Upon what does the amount of atmospheric resistance experienced by a moving body depend? (147)
185. Explain what is meant by this. (147)
186. What is the amount of atmospheric resistance experienced by a train of medium length moving at the rate of 10 miles per hour? (148)
187. If a body be moved along a surface without friction or atmospheric resistance, how may the units of work performed be found? (149)
188. When a train is moved along an inclined plane, how is the work performed by the locomotive found? (150)
189. Deduce a set of formulas for finding the horse power, weight, maximum speed, &c., of trains. (151)
190. What is meant by the modulus of a machine? (152)
191. Of machines for raising water, which has the greatest modulus? (153)
192. How may the work performed by water falling from a height be found? (154)
193. How is steam converted into a source of laboring force? (155)
194. What are the two principal varieties of the steam engine? (156)
195. What are the essential parts of the high pressure engine? (157)
196. How does the low pressure differ from the high pressure engine? (158
197. What are the varieties of the low pressure engine? (159)
198. How do these differ from each other? (160, 161)
199. In the high pressure engine, at what part of the stroke does atmospheric pressure act against the piston? (162)
200. Give the leading ideas that enter into the construction of the steam engine. (163)
201. In what respects is the low pressure engine preferable to the non-condensing engine? (164)

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202. How are the units of work performed by an engine found? (165)
203. Knowing the pressure of the steam on the boiler, how do we obtain the useful pressure on the piston? (166)
204. Give the rules for finding the H. P., &c., of engines. (167)
205. What is the real source of work in the steam engine? (168)
206. Why is it most advantageous to employ steam of high pressure? (168)
207. Give formulas for finding the area of the piston, length of stroke, pressure, effective evaporation, &c., in the steam engine. (169)
208. Define what is meant by a *fluid*. (171)
209. How is the term fluid commonly applied? (172)
210. Into what classes are fluids divided? Name the type of each. (173)
211. To what extent is water compressible? Alcohol? (173, Note)
212. How do liquids chiefly differ from gases? (174)
213. In what respects do liquids chiefly differ from solids? (175)
214. Give the most important consequences that flow from this fact. (175)
215. How would you illustrate the upward and lateral pressure of liquids? (175, Note)
216. What relation exists between the respective heights of two liquids of different densities placed in an inverted syphon? (176)
217. What is the amount of downward pressure exerted by a liquid confined in any vessel? (177)
218. How would you illustrate this fact? (177, Note.)
219. Show that weight and pressure are not to be confounded with one another. (177, Note 2.)
220. What are the weights respectively of a cubic inch, a cubic foot, and a gallon of water, at the temperature of 60° Fahr.? (178)
221. To what is the pressure exerted by water on a vertical or inclined surface equal? (179)
222. Give a rule for finding the lateral pressure exerted by water. (179)
223. How do you find the pressure exerted by water against a vertical or inclined surface at a given depth beneath the water? (180)

224. How do you find the pressure exerted against any fraction of a vertical surface when the upper edge is level with the surface of the water? (181)

225. Explain what is meant by transmission of pressure by liquids. (182)
226. Describe Bramah's Hydrostatic Press, and illustrate by a figure. (183)
227. Explain the principle upon which Bramah's Press acts. (182, Note)
228. For what purposes is Bramah's Press used? (184)
229. How do we find the relation between the power applied and the pressure obtaind by Bramah's Press? (185)
230. Describe what is meant by the hydrostatic paradox. (186)
231. Show that it is not in reality a paradox. (186, Note)
232. Describe the hydrostatic bellows. (187)
233. Give the rule for finding the upward pressure against the board of a hydrostatic bellows. (188)
234. When will a body float, sink, or rest in equilibrium in a fluid? (189)
235. What weight of liquid does a floating body displace? (190)
236. What portion of its weight is lost by a body immersed in a liquid? (191)
237. What is the specific gravity of a body? (192)
238. What is the standard of comparison for solids and liquids? (193)
239. What is the standard of comparison for all gases? (193)
240. How do we find the specific gravity of a so'id heavier than water? (194)
241. How do we find the specific gravity of a solid not sufficiently heavy to sink in water? (195)
242. What is the first method of finding the specific gravity of a liquid? (196)
243. What is the second method given for finding the specific gravity of a liquid? (196)
244. How is the specific gravity of a liquid determined by means of the hydrometer? (196)
245. Describe the hydrometer. (196)
246. What difference is there between hydrometers designed for determining the specific gravity of liquids specifically lighter than water, and those for ascertaining the specific gravity of liquids specifically heavier than water? (196)
247. How is the specific gravity of gases found? (197)
248. How may the weight of a cubic foot of any substance be found when its specific gravity is known? (199)
249. How may the solid contents of a body be found from its weight? (200)
250. How may the weight of a body be found from its solid contents? (201)

251. What is Pneumatics? (205)
252. What is the derivation of the word atmosphere? (206)
253. What is the atmosphere? (206)
254. To what height does the atmosphere extend? (207)
255. Give the exact composition of atmospheric air. (208)
256. What purpose is served by the oxygen in the air? (208, Note)
257. What purpose is served by the nitrogen? (208, Note)
258. Describe the principal properties of carbonic acid. (208, Note)
259. What are the chief sources of carbonic acid? (208, Note)
260. What is the maximum and what the minimum amount of carbonic acid in the air? (208, Note)
261. Describe the mode by which the air is kept sufficiently pure to sustain animal life. (208, Note)
262. Describe the property of gaseous diffusion. (209)
263. Explain how the property of gaseous diffusion affects the composition of the atmosphere. (209, Note)
264. Upon what does the amount of aqueous vapor present in the atmosphere depend? (210)
265. What is the maximum amount? What its minimum amount? (210)
266. To what is the blue color of the sky due? To what the golden tints of sunset? (211)
267. Which of the essential properties of matter belong to the air? (212)

268. How would you illustrate the impenetrability of the air ? (212, Note)
269. How would you illustrate the inertia of the air? (212, Note 2)
270. Why does air possess weight ? (213)
271. What may be taken as the fundamental fact of Pneumatics ? (213, Note)
272. What is the weight of 100 cubic inches of each of the following gases, viz., oxogen, hydrogen, nitrogen, atmospheric air, carbonic air? (213, Note)
273. Give some illustrations of the aggregate woight of the atmosphere (213, Note 2)
274. How is it that the lower strata of air are denser than the upper ? (214)
275. By what law does the density of the atmosphere decrease as we ascend ? (215)

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276. From what does the pressure of the air result? (216)
277. What do we mean by saying the pressure of the air is equal to 15 lbs. to the square inch ? (216, Note)
278. If the air were of the same density throughout, to what height would it extend? (217)
279. How is this known? (217)
280. How are permanently elastic gases chiefly distinguished from non-elastic gases? (219)
281. What is meant by permanently elastic gases? (219, Note)
282. Illustrate what is meant by the elasticity of a gas. (220)
283. To what is the elasticity of gases due ? (220, Note)
284. Enunciate Mariotte's law. (221)
285. Illustrate it by a bent tube as in Art. 218. (221, Note)
286. To what extent was Mariotte's law true? (222, Note)
287. What is the air-pump? (223)
288. By whom and when was it invented? (224, Note)
289. Describe the exhausting syringe. (225)
290. Draw a sketch of the air-pump, and describe its mode of action. (225, Note)
291. Upon what principle does the air-pump act? (226)
292. How perfect a vacuum can be secured by the air-pump? (226, Note)
293. Describe the condensing syringe. (227)
294. For what purpose is the air-pump chiefly used? (228)
295. Give some illustrations of the pressure of the air. (228, Note)
296. Give some illustrations of the elasticity of the air. (228 Note 2)

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297. What is the barometer? (229)
298. By whom and when was it invented? (229, Note)
299. What are the essential parts of a barometer? (230)
300. What is meant by the Torricellian vacuum? (230, Note)
301. How may the excellency of a barometer be tested? (231)
302. What is the cause of the oscillations of the barometer? (232)
303. In what regions of the earth are the oscillations of the barometer most fitful and extensive? (232, Note)
304. To what regular oscillations is the barometer subject? (233)
305. At what hours are the two maxima of pressure? (233)
306. At what hours are the two minima of pressure? (233)
307. In what region are the semi-diurnal oscillations greatest? (233, Note)
308. Give some idea of their extent in tropical countries, and explain why they are not observed in our climate. (233, Note)
309. How may the weather to be expected be foretold by the oscillations in the height of the barometric column? (234)
310. What does a fall in the barometer denote? (234, II.)
311. What does a rise in the barometer indicate? (234, III.)
312. What does a sudden change in the height of the mercury in the barometer denote? (234, IV.)
313. What does a steady rise in the column denote? (234, V.)
314. What does a steady fall in the column denote? (234, VI.)
315. What does a fluctuating state in the height of the column of mercury denote? (234, VII.)

316. Give Halley's rule for ascertaining the height of mountains, &c., by the barometer. (235)
317. Give Halley's rule with correction for temperature. (235)
318. Give Leslie's rule. (235)
319. Describe the essential parts of a common pump, and illustrate by a diagram. (236)
320. Explain why the common pump is sometimes called a lifting pump. (236, Note)
321. Explain the principle upon which the common pump acts. (236, Note)
322. Explain why the lower valve must be within 32 feet of the water in the reservoir in order that the pump may act at all times. (236, Note 2)
323. Describe the forcing pump. (237)
324. Describe the essential parts of a fire engine. (237, Note)
325. Describe the syphon. (238)
326. How is the syphon set in operation? (238, Note 1)
327. Explain upon what principle the syphon acts. (238, Note 2)

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328. When does the consideration of forces come under the science of statics? (239)
329. What kind of forces are considered in dynamics? (239)
330. Why is statics called a *deductive* science? (240)
331. Why is dynamics called an *inductive* or experimental science? (240)
332. What may force be defined to be? (241)
333. For what purposes is force required? (241)
334. What are the different kinds of forces as regards duration? (241)
335. What are the different kinds of continued forces? (242)
336. What may motion be defined to be? (243)
337. What are the qualities of motion? (244)
338. What are the different kinds of motion? (244)
339. What kind of a motion is produced by an accelerating, constant, or retarding force? (245)
340. What is velocity? (246)
341. Of how many kinds is velocity? (246)
342. When is velocity said to be uniform? (246)
343. What is momentum or motal force? (248)
344. To what are the momenta of bodies proportional? (249)
345. When the velocities of two moving bodies are equal, to what are their momenta proportional? (249)
346. When the masses of two moving bodies are equal, to what are their momenta proportional? (249)
347. When we speak of multiplying a velocity by a weight, what do we mean? (249, Note)
348. When force is communicated by impact to a body at rest, how long will the body remain at rest? (254)
349. Give the first general law of motion. (255)
350. Whose law is this? (257, Note)
351. Give the second law of motion. (256)
352. Whose law is this? (257, Note)
353. Give the third law of motion? (257)
354. Whose law is this? (257, Note)

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355. What is reflected motion? (258)
356. What is the angle of incidence? (258)
357. What is the angle of reflection? (258)
358. What proportion exists between the angle of incidence and the angle of reflection? (258)
359. How would all bodies fall in a vacuum? (259)
360. Upon what does the resistance encountered by a body moving through the atmosphere depend? (260)
361. What is the nature of the motion of a heavy body falling from a height? (261)

362. What velocity is acquired by a heavy body in falling through one second? (264)
363. Through how many feet does a body fall during the first second of its descent? (265)
364. Deduce a set of formulas for the descent of bodies freely through space. (266)
365. When a body is projected upwards what is the nature of its motion? (267)
366. Give the formulas for the motion of a body projected upwards or downwards? (268)
367. When a body is descending an incline how is the gravity expended? (269)
368. What are the laws of descent on inclined planes? (270)
369. Upon what is the final velocity of a body falling down an incline dependent? (271)
370. What are the laws of descent in curves? (273)
371. What is the brachystochrone?
372. What is the cycloid? (274)
373. Deduce a set of formulas for descent on inclines. (275, 276)

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374. What is a projectile? (277)
375. What forces influence projectiles? (278)
376. What is the theoretical path of a projectile? (278)
377. What is a parabola? (278, Note 1)
378. Upon what erroneous suppositions is the parabolic theory based? (278, Note 2)
379. Show that when a body is projected horizontally forward, the horizontal motion does not interfere with the action of gravity. (279, Note)
380. What are the three conclusions of the parabolic theory? (280)
381. What is the greatest horizontal range of a projectile? (280 Note)
382. To what is the velocity of projection speedily reduced, no matter what it may have been originally? (281)
383. How do you explain this? (281, Note 1)
384. What is the atmospheric resistance encountered by a ball or other projectile having a velocity of 2000 feet per second? (281, Note 2)
385. When a ball has considerable windage, what is the amount of deflection in its course? (281, Note 3)
386. What are the most important laws regarding the motion of projectiles thrown vertically into the air? (282)
387. What are the most important laws regarding the motion of projectiles thrown at an angle of elevation? (282)
388. To what is the explosive force of gunpowder exploded in a cannon equal? (283)
389. With what velocity does exploded gunpowder tend to expand? (283)
390. What is the composition of gunpowder? (283, Note)
391. What is the greatest initial velocity that can be given to a cannon ball? (284)
392. To what is the velocity of a ball of given weight fired with a given charge of powder proportional? (284, Note)
393. To what are the velocities of balls of equal weight fired by the same charge of powder proportional? (285)
394. To what are the velocities of balls of different weight but of the same dimensions fired by equal quantities of powder proportional? (286)
395. To what is the depth which a ball penetrates into an obstacle proportional? (287)
396. Give the rule for finding the velocity of any shot or shell when its weight and also that of the charge of powder are known? (288)

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397. What is centrifugal force? (289)
398. Why is it sometimes called tangential force? (289, Note)
399. What is centripetal force? (290)
400. When does a body move in a circle? (291)
401. When does a body move in an eclipse? (291)

402. How long can a rotating mass preserve itself? (292, Note 1)
403. Give some examples of the effects of centrifugal force (292, Note 2)
404. If the velocity and radius are constant, to what is the centrifugal force proportional? (293)
405. When the radius is constant, how does the centrifugal force vary? (294)
406. What is the amount of centrifugal force at the equator? (294, Note)
407. How rapidly must the earth revolve in order that the centrifugal force at the equator may equal gravity? (294, Note)
408. When the velocity is constant, how does the centrifugal force vary? (295)
409. When the number of revolutions is constant, to what is the centrifugal force proportional? (296)
410. Give a set of formulas for calculating centrifugal force. (297)
411. Give a rule for finding the work accumulated in a moving body. (299)
412. What is a pendulum? (300)
413. What is a simple pendulum? (301)
414. What is a compound or material pendulum? (302)
415. What is an oscillation or vibration? (303)
416. What is meant by the amplitude of the arc of vibration? (304)
417. What is meant by the duration of a vibration? (305)
418. What is meant by the length of a pendulum? (306)
419. What is the centre of suspension? (307)
420. What is the centre of oscillation? (308)
421. What is the centre of percussion? (308 Note)
422. What is meant by saying the centres of oscillation and suspension are interchangeable? (309)
423. How is the duration of a vibration affected by its amplitude? (310)
424. What is meant by saying the vibration of the pendulum is isochronous? (310, Note)
425. What relation exists between the lengths and times of vibrations of pendulums? (314)

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426. Give the chief laws of the oscillations of the pendulum? (311–316)
427. Why does the seconds pendulum vary in length in different latitudes? (316, Note)
428. What is the length of a seconds pendulum in Canada? (316, Note 2)
429. To what purposes is the pendulum applied? (317)
430. How is the pendulum used as a measure of time? (317, Note)
431. How is the pendulum used as a standard of measure? (317, Note)
432. How do we find the length of a pendulum to vibrate in a given time? (319)
433. How do we find the number of vibrations lost by a pendulum of given length when the force of gravity is decreased? (320)
434. How do we find the number of vibrations gained by a pendulum of given length when it is shortened? (321)
435. What is the science of Hydrodynamics? (322)
436. Enunciate Torricelli's theorem. (323)
437. In what time does a full vessel empty itself through an orifice in the bottom? (325)
438. How is the quantity of fluid discharged through an orifice of given size found? (326)
439. What is the *vena contracta*? (326, Note)
440. What relation exists between the theoretical discharge and the actual discharge? (326, Note)
441. Give the rule for finding the velocity and quantity of fluid discharged through an aperture of given size. (327)
442. What is an *adjutage*? (328)
443. Under what circumstance is the flow of water through an adjutage modified? (328)
444. How does a cylindrical adjutage increase the flow? (329)
445. How do conical adjutages increase the flow? (330, 331)
446. To what height does a jet of water, spouting upward from the bottom of a reservoir, reach?

447. When water spouts from an aperture in the side of a vessel, how is the horizontal distance to which it is thrown found? (333)
448. When a liquid flows through a pipe or channel, which part has the greatest velocity? (335)
449. How is the velocity of a stream determined? (336, Note 2)
450. What are the principal varieties of water wheels? (339)
451. In water wheels, when is the greatest mechanical effect produced? (340)
452. Give the rule for finding the horse powers of upright water wheels. (341)
453. What is a turbine wheel? How does it act? (342)
454. For what purposes are high and low pressure turbines respectively used? (343-5)
455. What are the principal advantages of the turbine over the upright water wheels? (346)

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456. What is the origin of all waves or undulations? (347)
457. Of how many kinds are undulations? (348)
458. What are progressive undulations? (349)
459. What are stationary undulations? (350)
460. What kinds of vibration may be imparted to a stretched string? (352)
461. What is meant by the *time of vibration*? (353)
462. What are the chief laws of the transverse vibration of cords? (354)
463. What are *nodal points*? (355)
464. What are the principal laws that govern the transverse vibrations of rods? (356)
465. How may an elastic plate be made to vibrate? (357)
466. What are nodal lines and nodal figures? (358-60)
467. What are the laws of vibration of elastic plates? (361)
468. Explain the cause and mode of undulation in liquids. (362)
469. Give the law of reflection of progressive undulations. (363)
470. Explain what is meant by the interference of waves and the phenomena resulting. (364)
471. Describe carefully the phonomena of undulations in an elastic fluid like the air. (366)

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472. What are the objects of the science of acoustics? (368)
473. What are sounds? (369)
474. Upon what does the intensity of a sound depend? (371)
475. How is the sound affected by the density of the medium in which it is produced? (372)
476. How does the *pitch* of a sound affect its velocity? (373)
477. How does the velocity of sound in atmospheric air vary? (374)
478. What is the velocity of sound in atmospheric air? (375)
479. Give the velocity of sound in several other media. (376, 377)
480. Upon what does the distance to which sound may be propagated depend? [378,379)
481. What is the result of the interference partial or complete, of sonorous waves? (380,381)
482. What laws govern the reflection of sound waves? (382,383)
483. What is an echo? (383)
484. What must be the least distance of the reflecting surface in order to produce a perfect echo? (384)
485. What are repeating echoes. (385)
486. Give some examples of remarkable echoes? (386)
487. Explain the construction of the so called whispering galleries. (387)
488. Name some of the best whispering galleries in the world. (387)
489. Describe the speaking trumpet and explain its mode of action. (388)
490. Describe the ear trumpet and explain the principle upon which it acts. (389)

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491. What is noise? (390)
492. What are musical sounds? (391)
493. What are the three elements of a sound? (392)
494. What is tone or pitch? Upon what does it depend? (393)
495. What is intensity? Upon what does it depend? (394)
496. What is the quality or timbre of a sound? (395)
497. When are sounds said to be in unison? (396)
498. What is a melody? (397)
499. What is a chord? (398)
500. What is harmony? (398)
501. Describe the siren and Savart's toothed wheel and explain their use. (400)
502. Describe the monochord and explain its use. (401)
503. Give the relative length of cords and the number of vibrations required to produce each note of the gamut. (402)
504. How is the absolute number of vibrations required to produce any given note determined? (403)
505. Give the number required for each of the notes of the common scale. (404)
506. How do we determine the number of vibrations required for the corresponding notes of higher or lower scales? (405)
507. How do we determine the length of a sonorous vibration? (406)
508. Give the lengths of the vibrations producing the C of different scales. (406)
509. What are intervals? (407)
510. How are musical intervals named? (408)
511. Give the fractional length of the interval between each two successive notes in the diatonic scale. (408 and 409)
512. What is a *major tone? minor tone? semitone?* (409)
513. What are diatonic and chromatic semitones? What is a *grave chromatic* semitone? What is a *comma?* (409, *Note*)
514. What are compound chords? (411)
515. What is the *perfect major accord?* (411)
516. What is the *perfect minor accord?* (411)
517. What is the difference between these as regards intervals? (411, *Note*)
518. Explain what is meant by the transposition of scales? (412)
519. What are *sharps* and *flats*, and for what purpose are they employed? (413)
520. What is the chromatic scale? (414)
521. What is temperament? (415)
522. Explain the use of temperament in music. (415)
523. Explain the phenomena of *beating* in musical sounds. (416)
524. Describe the diapason or tuning fork (417)

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525. Classify musical instruments. (418)
526. Describe the mode in which wind instruments are sounded, and name the most important wind instruments. (419)
527. Describe the mode in which stringed instruments are sounded. (420)
528. Describe the different varieties of the drum. (421)
529. Upon what circumstances does the pitch of the sound produced by a wind instrument depend? (422)
530. What causes the difference of timbre in wind instruments? (422)
531. What are the different modes of producing sounds in tubes? (423)
532. Give Bernoulli's laws governing the vibration of air in tubes. (424)
533. Name the several parts that constitute the organ of voice in man. (426)
534. Give the position and common name of the larynx. (427)
535. Describe the structure of larynx. (427)
536. What cartilages form the prominence known as Adam's apple in the front part of the throat? (427)
537. Where are the arytenoid cartilages placed? What is their use? (427)

538. Describe the cordæ vocales. What is their position and their attachments? (428)
539. What is the *rima glottidis?* What is its shape except during the production of sound? (428)
540. What is the glottis? What the *epiglottis*? (429)
541. Explain the production of sound in the larynx. (430)
542. Illustrate the extreme precision with which the will can determine the exact amount of tension of the vocal cords. (431)
543. Explain why a man sings base or tenor while women, girls and boys sing treble. (432)
544. How do you account for the difference of timbre in voices? (433)
545. Upon what does the loudness of the voice depend? (434)
546. How are voices divided by musicians? (435)
547. Give the extreme number of vibrations of each class of voices. (435)
548. What is the range of the voice in speaking? What is the range in singing? (435 *Note*)
549. Describe the production of sound in the inferior animals? (436)
550. What are the principal parts of the organ of hearing? (437)
551. Name and describe the two parts of the external ear. (438)
552. Name and describe the three parts of the middle ear. (439)
553. Name and describe the three parts of the internal ear. (440)
554. What is the fenestra ovalis? What the *fenestra rotunda*? (440)
555. Describe the position and probable use of the semi-circular canals? (440, 443)
556. How and where is the auditory nerve distributed in the ear? (441)
557. Describe the functions ot the different parts of the ear. (442)
558. What are the most grave and acute notes that are perceptible to the ear? (444)
559. Describe the mechanism of hearing in the different tribes of animals. (445)

THE END OF PART I.

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